EVALUATING ATTRIBUTE SIGNIFICANCE IN AHP USING SHANNON ENTROPY

Ewa van Uden, Miroslaw Kwiesielewicz
Faculty of Electrical and Control Engineering
Technical University of Gdansk
Narutowicza 11/12, 80-952 Gdansk – Poland
cotynato@ely.pg.gda.pl, mkwies@ely.pg.gda.pl

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Multi-attribute factor ranking in the sense of AHP is considered in this paper. It is assumed that in a multi-attribute decision-making process rankings of factors under particular attributes are given. Next, based on the Shannon entropy, an amount of information associated with each ranking is evaluated in order to specify an attribute with the smallest importance. Then a dimension of a decision problem solved can be reduced eliminating step by step less important attributes. The reduction of the problem dimension acquires importance when a number of attributes exceeds recommended in AHP size. The method also can be used to order the attributes. The approach proposed is illustrated by a numerical example.

Harker (1987) noticed that in a decision problem consisting of many alternatives and criteria, the number of necessary opinions becomes very large, for example, with 9 alternatives and 5 criteria a group of experts must answer 190 questions. In such cases, an expert is not always able to evaluate each pair of factors, particularly for all criteria. Harker proposed a method based on estimating missing data for such situations. Therefore a reduction of a decision problem dimension play a crucial role

The aim of this paper is to aid the decision-making process and reduce its complexity by qualifying the importance of each criterion. To avoid the pairwise comparisons of criteria, the quantity of information contained in each criterion is measured by Shannon entropy. In other words, an intention of proposed method is to simplify the process of giving weights to criteria, so that certain pairs of them need not be evaluated.

Let us assume, following (Sanchez and Soyer, 1998), that $\mathbf{p} = (p_1, \ldots, p_n)$ denotes a priority vector according to a certain criterion, after arithmetic normalisation (so that the vector's co-ordinates sum up to 1). Entropy for this vector may be defined as:

$$H(\mathbf{p}) = -\sum_{i=1}^{n} p_i \ln(p_i) \quad . \tag{1}$$

In information theory entropy H is defined as a measure of uncertainty of a discrete random variable X, which can take finite values $(x_1, \ldots, x_n)$ such that $P(X = x_i) = p_i$. In the AHP context, the priority p_i can be interpreted as the probability that the i –th alternative will be preferred by the decision-makerCriteria ranking may prompt the decision-maker as to what weights should be given to the criteria in the situation when his preferences are not specified precisely. For example, - the ranking vector of alternatives given in the form $\mathbf{v} = [1/n, \ldots, 1/n]$ does not provide any definite information – all alternatives are treated equally. When distinctiveness of alternatives increases, the entropy of such a vector decreases. Ranking of attributes from the point of view of the alternatives' distinctiveness could prompt the decision-maker to choose weights or particular attributes to ensure that the chosen decision differs fundamentally from the others. One can give low weights to those attributes that deliver the least information (by a uniform distribution of the alternatives' ranking vector) because their influence on the final vector is insignificant and there is no appreciable difference between alternatives. The ranking vector closer to uniform distribution means that the preferences between alternatives become indistinguishable.

Let us assume that a given decision problem consists of n possible alternatives $\mathbf{A_i}, i = 1,\ldots,n$ considered according to m criteria $\mathbf{K_j}, j = 1,\ldots,m$. As a result of the alternatives' pairwise comparisons with respect to particular criteria one obtains the ranking vectors concerning given criteria in the form:

$$\mathbf{v}_j = (v_{j1},\ldots,v_{jn}), \quad j = 1,\ldots,m \qquad . \tag{2}$$

Next, the aggregation according to criteria is done. The simplest method is a weighted sum normalised to one. In the original version of the method (Saaty, 1980) as well as for a fuzzy case (Laarhoven, Pedrycz, 1983) criteria are compared in pairs in order to evaluate their weights.

The problem arises, when the number of attributes becomes large and exceeds the recommended in AHP size (not more the 7 objects compared in the same time). The method described below allows order the attributes from the least to the most important. It can be a prompt for a decision-maker, which attributes (criteria) do not supply a meaningful amount of information.

Let us assume that weights $a_1,\ldots,a_m$ are unknown and satisfy the arithmetic normalisation condition:

$$\sum_{j=1}^{m} a_j = 1 \qquad , \tag{3}$$

and

$$a_j \geq 0 \quad \forall j \in 1,\ldots,m \qquad . \tag{4}$$

The result of aggregation is a final vector $\mathbf{v}$ in form:

$$\mathbf{v} = a_1\mathbf{v}_1 + \ldots + a_m\mathbf{v}_m \qquad . \tag{5}$$

The question posed by the authors is: for what values of $a_1,\ldots,a_m$ will the entropy of a vector $\mathbf{v}$ reach its minimum value? This question allows the following interpretation: for what values $a_1,\ldots,a_m$ will the decision-maker get the maximum amount of information.

The above considerations led to the following optimisation problem:

$$\min\left\{H(\mathbf{v}) = -\sum_{i=1}^{n} v_i \ln v_i\right\} \tag{6}$$

where:

$$v_i = \sum_{j=1}^{m} a_j v_{ji} \tag{7}$$

taking into an account conditions (3) and (4).

This paper presents an algorithm, based on the entropy measure, to determine the criterion that provides the greatest amount of information. Consequently multiple application of the proposed algorithm allows for criteria ranking as well as elimination the criteria with the small amount of information from decision process. Presented method can be particularly useful when the quantity of criteria is very big and the decision-maker would like to reduce some of them. The plans of developing this approach to fuzzy version of AHP, where rankings may be given in a quality form, using linguistic variables, is left for future.