A LOSS FUNCTION APPROACH TO GROUP PREFERENCE AGGREGATION IN THE AHP

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Summary: *The Analytic Hierarchy Process (AHP) is a useful method in aggregating group preference. We suggest a new method that uses consistency ratio as group evaluation quality. For this method, we introduce Taguchi's loss function. We also develop an evaluation reliability function to derive group weight. Lastly, we perform four experiments in order to confirm validity of this method.*

1. Introduction

The essentiality of multi-criteria decision making is to choose the best alternative from a set of competing alternatives that are evaluated under conflicting criteria. The Analytic Hierarchy Process as a multi-criteria decision making method provides us with a comprehensive framework for solving such decision problems by quantifying the subjective judgments.

The AHP as a growing field in both its theoretical and applied ramifications has been applied widely in decision making (Vargas, 1990; Zahedi, 1986; Shim, 1989). One of the topics on which research concentrates is the problem of group judgments aggregation and consistency ratio. The AHP has been criticized and enhanced on aggregation and consistency of individual judgments necessary for group decision making by many researchers (Aczel and Saaty, 1983, 1986, 1987; Forman, 1990; Forman and Peniwati, 1998; Ko and Lee, 2001; Lane and Verdini, 1989; Saaty and Mariano, 1979; Vargas, 1982).

However, they have focused mainly on the rules of aggregation such as geometric mean or arithmetic mean as well as consistency ratio itself. Thus, there was little research to use consistency ratio as evaluation quality of a group in aggregating group judgments.

The previous group aggregation methods keeps Saaty's rule that consistency ratio of individual pairwise comparison should be less than 0.1. That is, in aggregating individual judgments to a group opinion, one takes only individual judgments whose consistency ratio is less than 0.1. However, judgments are frequently inconsistent, and practically, pairwise comparison matrices rarely satisfy the consistency criterion (Murphy, 1993). Thus these methods tend to ignore many information of evaluation by using Saaty's consistency ratio. While previous methods dealt with weight of each evaluator, there have been no studies that tried to aggregate group priority by using weight of a group unit in aggregating individual judgment.

In order to overcome such problems, we introduce the concept of Taguchi's loss function (Antony and Kaye, 2000) and develop a loss function approach, which is a new method for improving group judgments aggregation. Taguchi defines quality as the loss. The smaller the loss, the higher the desirability. We define consistency ratio as the loss of evaluation quality. The loss of evaluation quality will be used as the weight of group in aggregating group judgments. We call this method the Weighted

After Geometric Mean Method (WAGMM) and the Weighted After Arithmetic Mean Method (WAAMM) for convenience.

2. Group Decision Making in the AHP

Group decision making involves weighted aggregation of different individual preferences to obtain a single collective preference. This subject has received a great deal of attention from researchers in many disciplines. Unfortunately, it is extremely difficult to accurately assess and quantify changing preferences, and to aggregate conflicting opinions held by diverse group.

2.1 Geometric Mean Method

Aczel et al. (1983, 1987) proposed a functional equation approach to aggregate the ratio judgments. Let us suppose that the numerical judgments $x_1, x_2, \cdots, x_m$ given by m persons lie in a continuum (interval) P of positive numbers so that P may contain $x_1, x_2, \cdots, x_m$ as well as their powers, reciprocals and geometric means, etc. The aggregating function will map Pm into a proper interval J, and $f(x_1, x_2, \cdots, x_m)$ will be called the result of the aggregation for the judgments $x_1, x_2, \cdots, x_m$. The function, which should satisfy the separability condition, unanimity condition and reciprocal condition, is the geometric mean as the following equation (1)

$$f(x_1, x_2, \cdots, x_m) = (x_1, x_2, \cdots, x_m)^{1/m} \tag{1}$$

Approach A: The approach A is to derive $\overline{A}$ from $\{A_i\}$. By applying equation (1) to every element of the pairwise comparison matrix $\{A_i\}$, we will have the following expression.

$$\overline{A} = \begin{bmatrix} \overline{a}_{11} & \overline{a}_{12} & \cdots & \overline{a}_{1n} \\ \overline{a}_{21} & \overline{a}_{22} & \cdots & \overline{a}_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \overline{a}_{n1} & \overline{a}_{n2} & \cdots & \overline{a}_{nn} \end{bmatrix} = \begin{bmatrix} \left(\prod_{i=1}^{m}\{\overline{a}_{11}\}_i\right)^{1/m} & \left(\prod_{i=1}^{m}\{\overline{a}_{12}\}_i\right)^{1/m} & \cdots & \left(\prod_{i=1}^{m}\{\overline{a}_{1n}\}_i\right)^{1/m} \\ \left(\prod_{i=1}^{m}\{\overline{a}_{21}\}_i\right)^{1/m} & \left(\prod_{i=1}^{m}\{\overline{a}_{22}\}_i\right)^{1/m} & \cdots & \left(\prod_{i=1}^{m}\{\overline{a}_{2n}\}_i\right)^{1/m} \\ \vdots & \vdots & \cdots & \vdots \\ \left(\prod_{i=1}^{m}\{\overline{a}_{n1}\}_i\right)^{1/m} & \left(\prod_{i=1}^{m}\{\overline{a}_{n2}\}_i\right)^{1/m} & \cdots & \left(\prod_{i=1}^{m}\{\overline{a}_{nn}\}_i\right)^{1/m} \end{bmatrix} \tag{2}$$

Once $\overline{A}$ is obtained, the priority $\overline{V}$ can be derived from $\overline{V} = f(\overline{A})$.

Approach B: As an alternative to approach A, the aggregated group priority vector $\overline{V}$ can be obtained from the priority vector of each person in the group. The priority vector fro each individual V_i of the group is derived from A_i.

$$V_i = f(A_i) = (\{v_1\}_i, \cdots, \{v_n\}_i), \quad i = 1,2,\cdots,m \tag{3}$$

$$\overline{V} = (\overline{v}_1, \overline{v}_2, \cdots, \overline{v}_n) = \left(\left(\prod_{i=1}^{m}\{v_1\}_i\right)^{1/m}, \left(\prod_{i=1}^{m}\{v_2\}_i\right)^{1/m}, \cdots, \left(\prod_{i=1}^{m}\{v_n\}_i\right)^{1/m} \right) \tag{4}$$

2.2 Arithmetic Mean Method

In addition to the geometric mean method discussed above, the arithmetic mean may also be used to aggregate group judgments. The only difference is that the arithmetic mean can only be applied on the final priority weights, i.e. Approach B. This is because of the reciprocal property of pairwise comparison and $1/\sum_{i=1}^{m} a_{jki} \neq \sum_{i=1}^{m} 1/a_{jki}$. The arithmetic mean method cannot be used to aggregate the pairwise comparison matrix A. So we have the mathematical form of the arithmetic mean operated on the priority weight as follows:

$$V_i = f(A_i) = (\{v_1\}_i, \cdots, \{v_n\}_i), \quad i = 1,2,\cdots,m \tag{5}$$

$$\overline{V} = (\bar{v}_1, \bar{v}_2, \cdots, \bar{v}_n) = \left(\frac{1}{m} \sum_{i=1}^{m} \{v_1\}_i, \frac{1}{m} \sum_{i=1}^{m} \{v_2\}_i, \cdots, \frac{1}{m} \sum_{i=1}^{m} \{v_n\}_i \right) \tag{6}$$

3. Loss Function Approach

3.1 Loss Function

Loss function has three types of characteristics such as nominal-is- best characteristics, Smaller-is-better characteristics and Large-is-better characteristics.

3.1.1 Nominal-is-best Characteristics

Taguchi suggests a quadratic loss function. Taguchi's loss function shows that as a characteristic deviates or moves further away from its target value, an increasing loss will be incurred. The smaller the characteristic variation about the target value, the better.
Let y be a characteristic and let m denote the target value. Taylor series expansion loss function $L(y)$ for m is given by equation (7).

$$L(y) = L(m) + L'(m)(y-m) + \frac{L''(m)}{2}(y-m)^2 + \cdots \tag{7}$$

As shown in Fig1 (a),

$$L(m) = 0, \quad L'(m) = 0 \tag{8}$$

Thus, if one ignore terms of third order, then loss function for nominal-is-best characteristics is given by equation (9)

$$L(y) = k(y-m)^2 \tag{9}$$

where k is $L''(m)/2$. Let A be loss of customer when tolerance limit is $m + \Delta$.

$$k = \frac{A}{\Delta^2} \tag{10}$$

where Δ is the distance from the target m to a tolerance limit and A is the cost when characteristic exceeds the tolerance limits (i.e. $m + \Delta$). The loss function for this characteristic is shown in Fig 1(a).
Let $E(y), V(y)$ be m, s^2, respectively. The average loss can be obtained by equation (11).

$$\begin{aligned} L &= E[L(y)] = E[k(y-m)^2] = kE(y-m)^2 \\ &= kE\{[y - E(y)] + [E(y) - m]\}^2 = k[s^2 + (m-m)^2] \end{aligned} \tag{11}$$

3.1.2 Smaller-is-better Characteristics

In this case, the characteristic y is continuous and positive with the most desired value of zero. Here the loss function $L(y)$ increases as y increases from zero. The loss function for smaller-is-better characteristics is given by equation (12).

$$L(y) = ky^2, \ k = \frac{A}{\Delta^2} \tag{12}$$

Because y is continuous and positive, the loss function $L(y)$ is a one-sided function and therefore cannot accepts negative values. The loss junction for this characteristic is shown in Fig 1(b).
The average loss can be obtained by equation (13).

$$L = kE(y^2) = k(s^2 + m^2) \tag{13}$$

3.1.3 Large-is-better Characteristics

In this case, the characteristic is continuous where one would like the characteristic to be as large as possible. The loss function $L(y)$ becomes progressively smaller as the value of the characteristic y

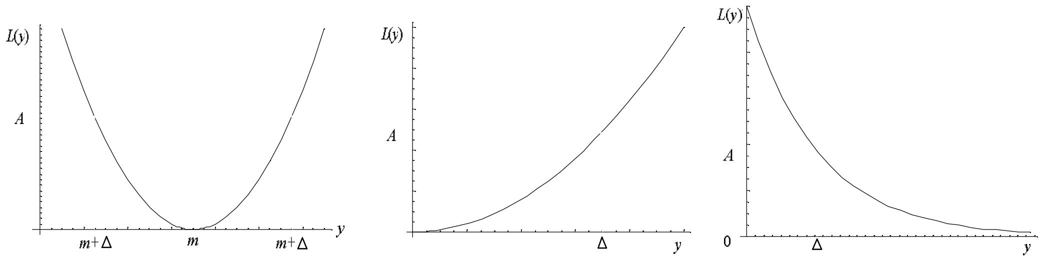
increases along the x-axis. The ideal value of this type of characteristic is infinity and the loss at that point is zero. The loss function for large-is-better characteristics is given by equation (14).

$$L(y) = k\left(\frac{1}{y^2}\right) \tag{14}$$

where the loss coefficient k is determined by equation (15).

$$k = A_0\Delta_0^2 \tag{15}$$

The loss function for large-is-better characteristics is shown in Fig 1(c).



$$F(X) = 1 \qquad X = 0$$
$$F(X) = Exp(-aX) \quad 0 < X < \text{Tolerance limit} \tag{17}$$
$$F(X) = 0 \qquad X \geq \text{Tolerance limit}$$

where, a is coefficient by each dimension.
The characteristic of this function is that the greater the expected loss of the group, the smaller the group weight.

Many different types of 'evaluation reliability function' can be defined according to the dimension of pairwise comparison matrix. The tolerance limits are required to define the function. Thus, we use the square of Ko's consistency ratio (Ko and Lee, 2001) as tolerance limits for 'evaluation reliability function'. Tolerance limits of evaluation reliability function are shown in Table 1.

Table 1. Tolerance limits of evaluation reliability function

Dimension	Tolerance Limit
3	0 < X < 0.0049
4	0 < X < 0.0053
5	0 < X < 0.1369
6	0 < X < 0.3136
7	0 < X < 0.4489

When an expected loss for consistency ratio of a group becomes 0, the weight of the group has a value of 1. When an expected loss is beyond tolerance limit, the group has a value of 0.

3.3 The Procedure

Step1. Compute $\overline{CR}$ and V_{CR} as mean and variance of consistency ratio for pairwise comparison judgments of each individual evaluator as shown the following equation (18).

$$\overline{CR} = \sum_{i=1}^{n} \frac{CR_i}{n}, \qquad V_{CR} = \sum_{i=1}^{n} \frac{(CR_i - \overline{CR})^2}{(n-1)} \tag{18}$$

where, CR_i is consistency ratio for evaluator i ($i = 1,2,\cdots,n$), $\overline{CR}$ is mean of consistency ratio, V_{CR} is variance of consistency ratio and n is the number of evaluator.

Step2. Calculate an expected loss estimate X of consistency ratio from mean and variance obtained in Step1. An expected loss estimate is given by the following equation (19).

$$X = E(L) = V_{CR} + (\overline{CR})^2 \tag{19}$$

where, $E(L)$ is an expected loss estimate of consistency ratio.

Step3. Calculate a group weight from evaluation reliability function. A graph and equation of dimension of evaluation reliability function are as follows:

Table 2. Evaluation reliability function

n	Evaluation reliability function	n	Evaluation reliability function
3	$F(X)=1 \quad X=0$ $F(X)=Exp(-750X) \quad 0<X<0.0049$ (20) $F(X)=0 \quad X \geq 0.0049$	4	$F(X)=1 \quad X=0$ $F(X)=Exp(-70X) \quad 0<X<0.0529$ (21) $F(X)=0 \quad X \geq 0.0529$
5	$F(X)=1 \quad X=0$ $F(X)=Exp(-27X) \quad 0<X<0.1369$ (22) $F(X)=0 \quad X \geq 0.1369$	6	$F(X)=1 \quad X=0$ $F(X)=Exp(-10X) \quad 0<X<0.3136$ (23) $F(X)=0 \quad X \geq 0.3136$
7			$F(X)=1 \quad X=0$ $F(X)=Exp(-8X) \quad 0<X<0.4489$ $F(X)=0 \quad X \geq 0.4489$

(24)

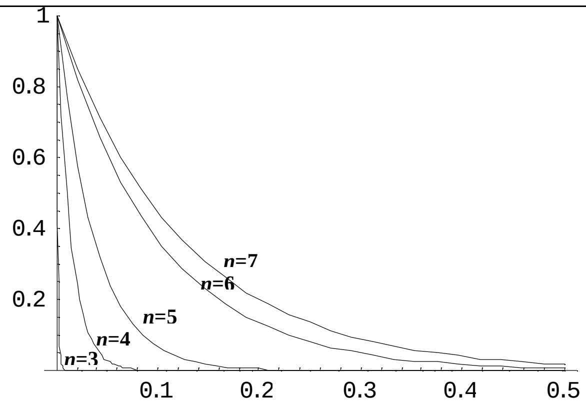


Fig. 2. Evaluation Reliability Function

Step4. Aggregate group priorities to consider an evaluation quality.

$$A_i = a \sum_{j=1}^{n} F(X_j) A_{ij} C_j \tag{25}$$

where, A_i is total priority ($i = 1,2,\cdots,m$), $F(X_j)$ is jth group weight ($j = 1,2,\cdots,n$), A_{ij} is priority for alternatives, C_j is priority for criteria, i is the number of alternative, j is the number of criterion and a is normalizing constant.

3.4 Experimental Design

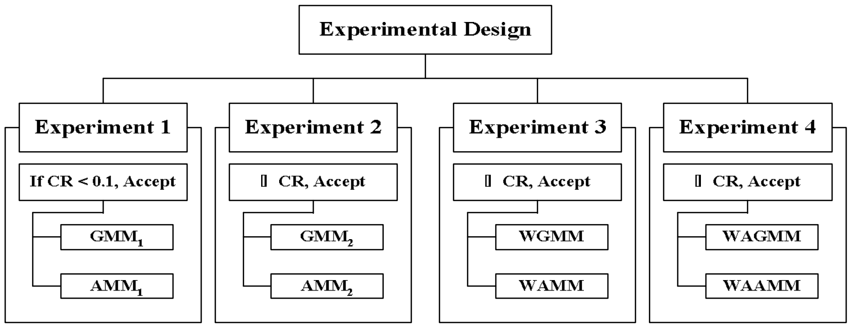
We performed four experiments in order to confirm validity of this approach.

Experiment 1(GMM$_1$ and AMM$_1$): we use geometric mean method (GMM$_1$) and arithmetic mean method (AMM$_1$) for aggregating pairwise comparison judgments. In experiment 1, we take only judgments of evaluators whose consistency ratio pairwise comparison matrix is less than 0.1.

Experiment 2(GMM$_2$ and AMM$_2$): we also use geometric mean method (GMM$_2$) and arithmetic mean method (AMM$_2$) for aggregating pairwise comparison judgments. However, in experiment 2, any consistency ratio would be accepted.

Experiment 3(WGMM and WAMM): In experiment 3, we use weighted geometric mean method (WGMM) and weighted arithmetic mean method (WAMM) for aggregating pairwise comparison judgments. We use consistency ratio as a weight of individual evaluator, which is calculated by Kim's method (Kim and Eo, 1994). Experiment3 also accept any consistency ratio.

Experiment 4(A loss function approach): We define WAGMM as weighted after geometric mean method and WAAMM as weighted after arithmetic mean method. We use consistency ratio as a weight of evaluators. Weights are calculated by a new method. Experiment4 also accept any consistency ratio.


Experimental Design
Experiment 1
If CR < 0.1, Accept
GMM1
AMM1
Experiment 2
CR, Accept
GMM2
AMM2
Experiment 3
CR, Accept
WGMM
WAMM
Experiment 4
CR, Accept
WAGMM
WAAMM

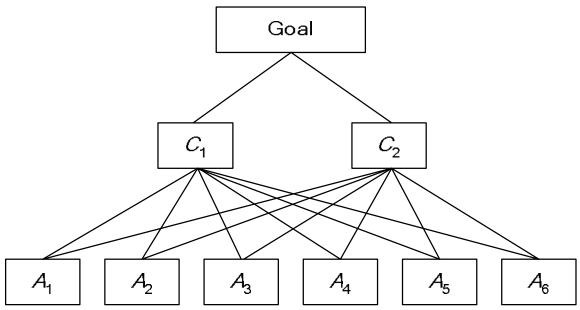

Goal
C1
C2
A1
A2
A3
A4
A5
A6

Table 5. Priority and rank for the alternatives using the GMM_2 and AMM_2

Alt.	Priority(GMM_2)		Total priority	Rank	Alt	Priority(AMM_2)		Total priority	Rank
	C_1	C_2				C_1	C_2		
A_1	0.441	0.216	0.3285	1	A_1	0.403	0.213	0.3094	1
A_2	0.133	0.318	0.2255	2	A_2	0.142	0.314	0.2277	2
A_3	0.163	0.176	0.1695	3	A_3	0.173	0.177	0.1749	3
A_4	0.081	0.045	0.0630	6	A_4	0.082	0.045	0.0636	6
A_5	0.082	0.204	0.1430	4	A_5	0.090	0.201	0.1455	4
A_6	0.101	0.041	0.0710	5	A_6	0.110	0.047	0.0787	5

Table 6. Weight of each individual evaluator

	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9	E_{10}
For C1	0.0355	0.1357	0.1530	0.2032	0.2413	0.1357	0.0228	0.0190	0.0217	0.0221
For C2	0.0284	0.1673	0.1673	0.1249	0.0997	0.0997	0.0718	0.0121	0.0997	0.1249

Table 7. Priority and rank for the alternatives using the WGMM and WAMM

Alt.	Priority(WGMM)		Total priority	Rank	Alt	Priority(WAMM)		Total priority	Rank
	C_1	C_2				C_1	C_2		
A_1	0.4045	0.2143	0.3094	1	A_1	0.400	0.212	0.3062	1
A_2	0.1596	0.3235	0.2416	2	A_2	0.167	0.321	0.2441	2
A_3	0.1689	0.1851	0.1770	3	A_3	0.174	0.184	0.1789	3
A_4	0.0851	0.0445	0.0648	5	A_4	0.088	0.045	0.0664	5
A_5	0.0729	0.1967	0.1348	4	A_5	0.072	0.195	0.1337	4
A_6	0.0839	0.0379	0.0609	6	A_6	0.090	0.038	0.0639	6

Table 8. Weight of an evaluation quality

	Mean	Variance	Expected loss, X_i	$F(X_i)$
1	0.323	0.076401	0.18073	0.16
2	0.090	0.025997	0.02598	0.77

Table 9. Priority and rank for the alternatives using the WAGMM and WAAMM

Alt.	Pri.(WAGMM)		Total priority	Nor.	rank	Alt	Pri.(WAAMM)		Total priority	Nor.	rank
	C_1	C_2					C_1	C_2			
A_1	0.441	0.216	0.1184	0.2547	2	A_1	0.403	0.216	0.1153	0.2431	2
A_2	0.133	0.318	0.1331	0.2861	1	A_2	0.142	0.314	0.1321	0.2841	1
A_3	0.163	0.176	0.0808	0.1737	4	A_3	0.173	0.177	0.0819	0.1763	4
A_4	0.081	0.045	0.0238	0.0512	6	A_4	0.082	0.045	0.2392	0.0515	6
A_5	0.082	0.204	0.0851	0.1830	3	A_5	0.090	0.201	0.0846	0.1821	3
A_6	0.101	0.041	0.0239	0.0513	5	A_6	0.110	0.047	0.0270	0.0599	5

4.3 Findings

The findings are classified into five categories through comparison of results. Comparison of the result is summarized in Table 10 and 11.

GMM versus AMM: Our experimental results show that the rank of geometric mean method and arithmetic mean method commonly is not different. It is certain that the ranks of GMM_1 and AMM_1 are same as mentioned Aczel and Saaty (1983, 1986, 1987). The other comparisons confirmed us that GMM and AMM are not different in aggregating individual judgments.

Application of CR versus Not Application of CR: The comparison results for GMM_1 (AMM_1) and GMM_2 (AMM_2) are little different. These results indicate that methods by Saaty and the other method are not different. That is, in aggregating individual judgments to a group opinion, consistency ratio does not impact on group priority.

GMM (AMM) versus WGMM (WAMM): The ranks GMM (AMM) and WGMM (WAMM) are the same. Individual weight by consistency ratio does not impact on group priority. Particularly, the more number of evaluator, the lower impact on weight.

GMM (AMM) versus WAGMM (WAAMM): The rank of GMM (AMM) differs from the rank of WAGMM (WAAMM). This result indicates that a loss function approach is appropriate for aggregating individual judgments.

WGMM (WAMM) versus WAGMM (WAAMM): The comparison results for WGMM (WAMM) and WAGMM (WAAMM) are different. While the weight for each evaluator is calculated in WGMM (WAMM), the weight for a group is calculated in WAGMM (WAAMM). This result indicates that the difference is occurred by a way to derive weight. It is proved that loss function approach is better method than others.

Table 10. Comparison of geometric mean methods

Alt.	GMM_1		GMM_2		WGMM		WAGMM	
	priority	rank	Priority	rank	priority	rank	priority	Rank
A_1	0.3105	1	0.3285	1	0.3094	1	0.2547	2
A_2	0.2510	2	0.2255	2	0.2416	2	0.2861	1
A_3	0.1785	3	0.1695	3	0.1770	3	0.1737	4
A_4	0.0685	5	0.0630	6	0.0648	5	0.0512	6
A_5	0.1305	4	0.1430	4	0.1348	4	0.1830	3
A_6	0.0605	6	0.0710	5	0.0609	6	0.0513	5

Table 11. Comparison of arithmetic mean methods

Alt.	AMM_1		AMM_2		WAMM		WAAMM	
	priority	rank	Priority	rank	priority	rank	priority	Rank
A_1	0.3058	1	0.3094	1	0.3062	1	0.2481	2
A_2	0.2530	2	0.2277	2	0.2441	2	0.2841	1
A_3	0.1804	3	0.1749	3	0.1789	3	0.1763	4
A_4	0.0695	5	0.0636	6	0.0664	5	0.0515	6
A_5	0.1282	4	0.1455	4	0.1337	4	0.1821	3
A_6	0.0632	6	0.0787	5	0.0639	6	0.0599	5

5. Conclusion

This study has developed a new method to derive group priority. For group decisions, we used an expected loss of a group and evaluation reliability function. As shown above, this process of proposed method is very simple and appropriated to derive group priority.

This method has some limitation. First of all, the loss function approach proposed in this study is not applied to the highest level of a hierarchy. Thus, it is necessary to develop a way to be applied to the highest level. We only proposed Evaluation Reliability Function for pairwise comparison matrix from three dimensions to seven dimensions. However, it is necessary to extend it to eight dimensions and nine dimensions. We made an evaluation reliability function into exponential function. It is necessary to develop various form of evaluation reliability function according to characteristic of a group.

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