AHP FOR GROUP DECISION-MAKING WITH OPINION MODIFYING PROCESS

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Summary: *In this paper, we propose a method for group decision-making using the AHP concept. While some studies exist for group decision-making using the AHP concept, almost all of them use the static methods. However, there are many situations where we need to discuss repeatedly until an agreement is obtained. In this paper, we focus on opinion modification in discussion processes and propose a method for obtaining agreement in a group using interactive discussion. Our method is divided into three processes: preparation, opinion declaration and opinion modification. Moreover, to modify opinions, we show quantitative information on the effect of change in term of the pairwise value. By showing the information for modifying opinions and making the criterion for agreement explicit, the members who participated in our experimental example were able to reach a satisfying.*

1. Introduction

There are many cases where decisions are made in a group. In such a case, each member who belongs to the group often has a different opinion or criterion for achieving the goal. To integrate these opinions, we often discuss repeatedly or persuade the other members to make an integrated conclusion. AHP works as a powerful tool for decision-making by settling the problem and making the evaluation criteria clear. We also believe AHP is a useful method in the case of group decision-making, but it needs to integrate many different opinions, which can be very difficult. When several members discuss a problem, opinions are often different at first. In such a case, the members repeat the discussions and declare modified opinions, which leads to a convergence of opinion. So, when we utilize AHP for group decision-making, we need to determine the way to converge the opinion. In order to do this, it is important to effectively show the information for modifying the opinions.

2. Previous Studies

Some methods have proposed group-decision-making using the AHP concept. Saaty (1980) adopted the geometric mean of each term of each matrix as the pairwise value for the integrated opinion in the group. This method is broadly utilized for group-decision-making using AHP, but may create a result far from all members' opinions. Yamada et al. (1997) proposed "interval AHP" for group-decision-making. In their method, each member shows the interval for each term of each matrix that can easily be

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accepted as his/her opinion. The union is the integrated opinion if there is union on intervals, and if there is no union set, the maximum range of all intervals is the integrated pairwise value. Then, they solve the optimal weight by the least square method. Elsewhere, Nakanishi et al. (1998) discussed a method based on ranking for each member. In this sense, their method is an application of Yamada et al.'s method, and they integrate the value by least square method using the weight for each member.

These methods are very useful when all members can accept the first calculated solution. But the weights or values of pairwise comparison need to be modified when the members disagree on the first calculated solution. Kato et al. (1997) have proposed a method for group decision-making using AHP and built a computer-based DSS system. Their method uses interactive group decision-making. In this method, each member indicates the degree of demand on other opinions, subjectively. Takano and Suzuki (2002) have used cluster analysis to make groups whose members have the same opinion. Then, the arithmetic mean in the group is regarded as the opinion of the group, and they have proposed a way to match the evaluations of alternatives. However, in many group decision-making situations, members discuss the factor of disagreement and modify the evaluation for the factor. In such a case, the decision-making should not be represented a rough consensus, but should be pursued tenaciously by each member.

In this study, we propose a method for group decision-making using the AHP concept. We focus especially on the opinion modifying process in the overall decision-making process and propose a method for obtaining agreement in a group using *interactive discussion.*

3. Proposed Method

In our proposed method, we focus on the next three characteristics. First, we consider the opinion modifying process during discussion. Second, we adopt an opinion modifying method where every member can be convinced. Third, the direction for modifying members' opinions is considered. In subsection 3.1, we explain these three characteristics in detail. In 3.2, we outline the procedure used in the proposed.

3.1 Characteristics of the Proposed Method

3.1.1 Opinion Modifying Process

Most previous studies have been aimed at integrating the different opinions into unique pairwise comparison values. In the group-decision-making in this study, we assume a case where discussions among members are possible. In such a situation, we also consider two repeating phases where members discuss and modify their opinions. This study aims to propose a method such "a repetition of *discussion* and *opinion modifying*." Some previous studies have also pointed out that the interactive process is important when agreement cannot be obtained in one step. Some interactive methods for group decision-making have been proposed, but they do not consider the case where members discuss directly. As mentioned above, it is important that all members are convinced by the final conclusion, so a process to modify each opinion after hearing other members' opinions is required.

3.1.2 Convinced Modifying Method

In this study, the following four items are assumed concerning how opinions are modified.

- If a member agrees with the opinion of another member, the member modifies his/her opinion in the direction of the other member's opinion (composite weights vector).
- If a member does not agree with all the other opinions, he/she maintains his/her opinions.
- If a member agrees with more than one other opinion, the member modifies his/her opinion in the direction of the other members' opinions. In this case, the degrees of conviction for

these members are considered using the weights of composite vector for each member, after which we obtain a weighted average to decide the direction.
- Each member shows his/her degree of conviction as the degree of support for his/her opinion.

We call this method "the convinced modifying method" when each member modifies his/her opinion under the above assumptions. Figure 1 shows the concept of the convinced modifying method.

Figure 1 illustrates the case of 3 alternatives and 4 members. The outside equilateral triangle is 3-dimensional space, which comprises each element of composite weight coordinates. Let $\mathbf{w}^d$ be the composite weight for member *d*. $\mathbf{w}^d$ is placed in the equilateral triangle because the sum of elements of $\mathbf{w}^d$ is normalized into unity. Here, let $\mathbf{r}^d$ be the modifying direction for member *d* vector, and $\mathbf{w}^d$ be the coordinate weights vector. The double circle and black dots indicate the coordinate of composite weights of the object member and the other members.

Now, we consider the case that member *d* changes his/her opinion from $\mathbf{w}^d$. Based on the above assumptions, to modify the opinion means placing the direction of modified opinion into the colored convex region with respect to the coordinates of all opinions. The direction $\mathbf{r}^d$ is determined by the weighted average of the degree of conviction for all members included his/herself. Note that this method only determines the direction but cannot show the amount of modification, so the modified composite weight may pass out of the shadowed region.

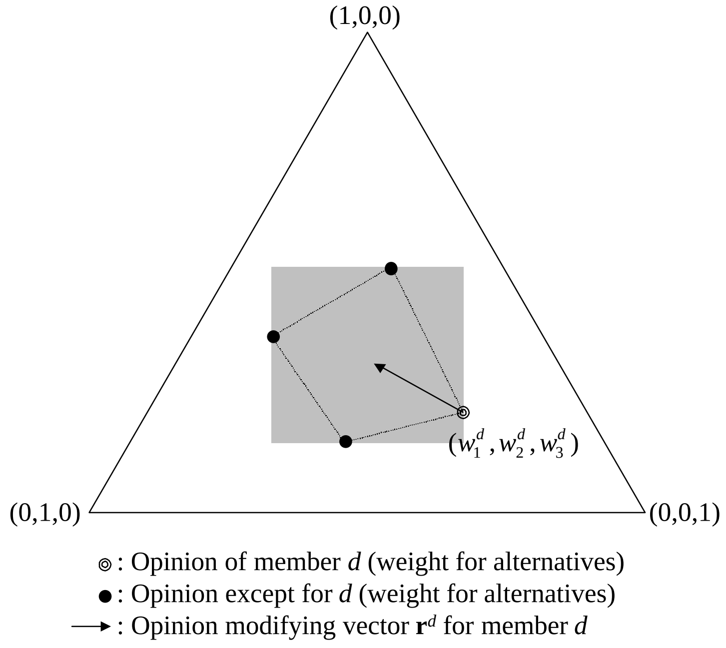


Figure 1. Concept of convinced modifying method (the case of 3 alternatives and 4 members)

3.1.3 Opinion Modifying Process

In AHP, we input the pairwise comparison values and obtain weights as output. So, modifying pairwise comparison values should also create opinion modifying. To do this, we use the method of Kato et al. (Kato et al., 1997), explained below..

AHP evaluates plural alternatives through the weights vector **w** (output), which is obtained from the pairwise comparison values a_{ij} (output). Therefore, when we modify the evaluation, we should not change the weights directly, but we change the pairwise comparison values and re-calculate the weights. However, it is hard to identify at a glance how the evaluation is modified effectively. So, a repeated process of trial and error is usually required. The same also applied to the consistency index (C.I.).

To resolve this situation, Masuda (1987) has proposed sensitivity coefficients for C.I. and **w** with respect to a_{ij},

$$\partial \text{C.I.}/\partial a_{ij}, \ \partial \mathbf{w}/\partial a_{ij}. \tag{1}$$

He has also shown that the evaluation can be modified effectively using equation (1).

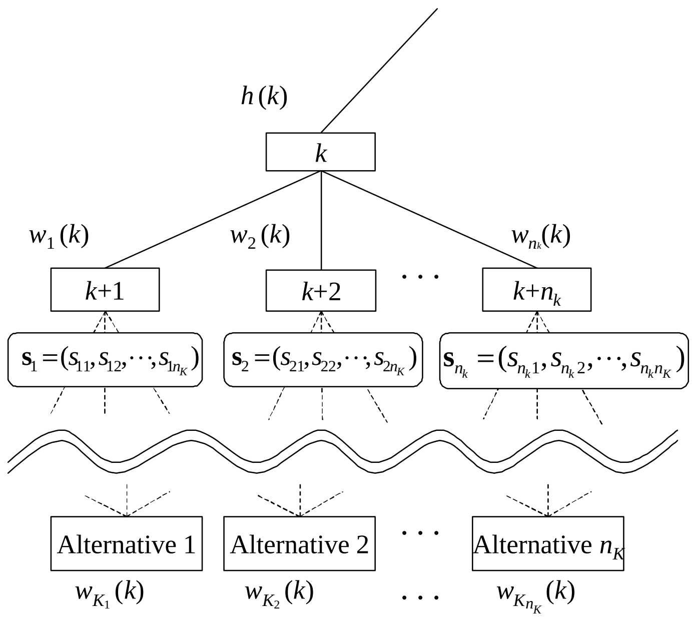
Kato et al. has proposed a method to determine the direction for modification by using equation (1). By calculating the degree of effectiveness of modification for pairwise (i, j), we can determine the effective direction of modifying, and we can modify the value of a_{ij} that is the highest effectiveness of modification. They call this the "opinion modifying support method." Next, we explain their method in detail.

Their method solves which $a_{ij}(k)$ among n_k elements under a criterion k the most effective element of modification can be made. Specifically, we calculate the degree of effectiveness of modification $g_{ij}(k)$. The effect is as large as the value of $g_{ij}(k)$. $g_{ij}(k)$ is shown in the next equation,

$$g_{ij}(k) = h(k)\, \mathbf{w}_{ij}(k)\, S(k)\, \mathbf{r}(k). \tag{2}$$

where $h(k)$ is the weight given by the decision-maker to the k-th criterion in any level, $\mathbf{w}(k)$ is the degree of the important vector ($w_1(k)$, $w_2(k)$,.., $w_{n_k}(k)$), $\mathbf{w}_{ij}(k) = \partial \mathbf{w}(k) / \partial a_{ij}(k)$ is the n_k-dimensional row vector. This means the sensitivity coefficient for the change of $\mathbf{w}(k)$ with respect to $a_{ij}(k)$, $\mathbf{S}(k)$ is the $n_k \times n_k$ matrix arranged horizontally $\mathbf{s}_{1(k)}$, $\mathbf{s}_{2(k)}$,…, $\mathbf{s}_{nK(k)}$, which is the weight vector of n_K alternatives for each n_k elements depending directly under the k-th criterion, and $\mathbf{r}(k)$ is the n_K -dimensional column vector that shows the intensity of the required increase or decrease in the degree of importance of n_K alternatives from the k-th criterion viewpoint. When the element of $\mathbf{r}(k)$ is positive or negative, then it indicates the requirement is respectively, an increase or a decrease.

Figure 3 shows the relationship among the above variables. Namely, $g_{ij}(k)$ with respect to $a_{ij}(k)$ is the degree of modification in the direction which the weight of alternatives is required when we attach greater importance to element j than to i,. So, when $g_{ij}(k)$ is positive or negative, then the effectiveness is



positive or negative, respectively. It remains a problem how $\mathbf{r}(k)$ is determined.

Figure 2. The relationship among the variables for opinion modifying support method (Kato et al., 2000)

In this study, the vector $\mathbf{r}^d(k)$, which is explained in the previous subsection, is utilized for $\mathbf{r}(k)$ in equation (1). Therefore, the degree of effectiveness of modification $g_{ij}(k)$ is obtained for each member as $g_{ij}^{d}(k)$ (d=1,2,…,m).

However, as mentioned in the previous subsection, $\mathbf{r}^d$ only shows the direction, so it cannot determine the amount of modification. This method also points out the pairwise comparison value for effective modification and shows the direction, but cannot obtain the amount of required change.

3.2 Procedure for the Proposed Method

Figure 3 shows the flowchart for the proposed method. The method is divided into three broad parts: *preparation*, *opinion declaration* and *opinion modifying*.

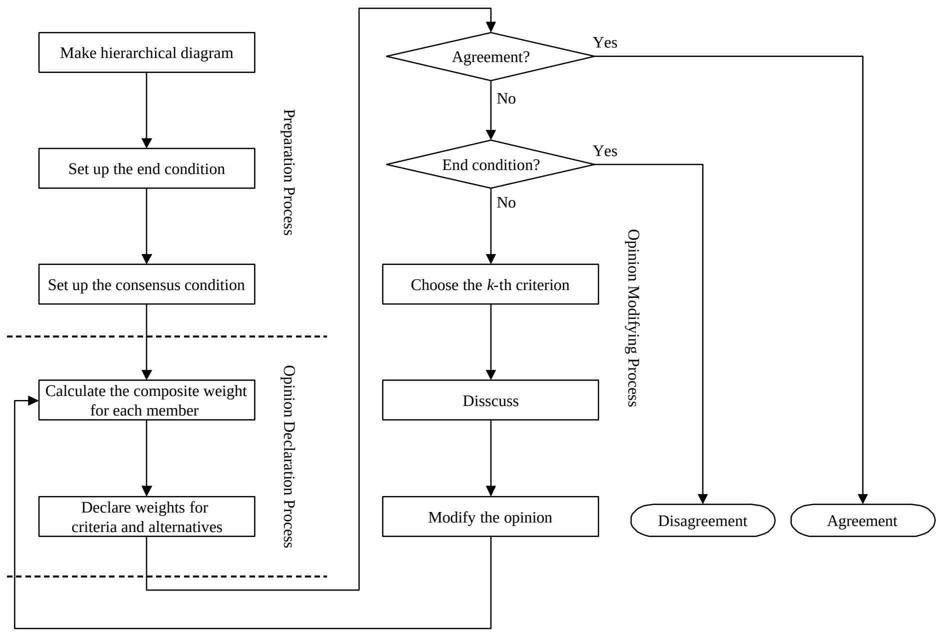


Figure 3. Flowchart for the proposed method

3.2.1 Preparation Process

Making a hierarchal structure diagram

If the hierarchal diagram has not been prepared yet, the members must create one. This can be achieved through the brainstorming or ISM, and so on.

Setting up the end condition and achievement of agreement

Next, we set up the end condition and the achievement of agreement. Theoretically, the opinions should be modified repeatedly until agreement is obtained. However, in actually, this is impossible. Therefore, the condition for terminating discussion should be given in advance, e.g. a time limit or the number of modifications. The condition for achieving agreement should also be set, which enables members to judge whether agreement has been reached. Generally speaking, the following two patterns prevail.

- The alternative ranked first by all members is the same.
- Overall ranking is unanimous.

Agreement by all members is required for the conditions to obtain.

3.2.2 Opinion Declaration Process

Calculating the composite weight for each alternative of each member

In this process, each member declares the composite weight for each alternative using AHP. If the C. I. value is outside the acceptable range, e.g. over 0.1, then we demand a modification of opinion. For modifying the pairwise comparison value to improve the C.I. value, we adopt Masuda's method. Comparing the value of $\partial \text{C.I.}/\partial a_{ij}$, we demand the modification of the pairwise comparison value corresponding with the maximum value.

Declaring the individual and composite weights

After calculating the composite weights, each member declares the value as his/her opinion to the other members. At this time, individual weights are also shown to reference.

3.3.3 Opinion Modifying Process

Judging the agreement

Based on the declared composite weights from the members, if the agreement condition is satisfied, then we have a convinced conclusion and the procedure ends.

Judging the end without agreement

If the agreement condition is not satisfied but the end condition as noticed as noted in the previous paragraph is satisfied, the discussion is terminated without agreement. If the procedure ends without agreement, it should be restarted from the first step, i.e. making the hierarchical diagram.

Choosing the criterion for discussion

When agreement is not obtained, we proceed to the discussion. Before discussion, the criterion that is the object of discussion must be chosen. The method for selecting a criterion is not the same in various decision-making scenes. A criterion from which opinions seldom differs may be chosen and the members converge little by little. Alternatively, a criterion to which opinions are totally opposed may be chosen.

Discussing

Next, comparing own opinions to other opinions, opinions are freely exchanged among members. However, endless talking will not improve the effect, so a definite time limit should be set in advance.

Modifying opinions

When the discussion is closed, then each member modifies his/her opinion based on the convinced modifying method explained in subsection 3.1.2.

First, each member expresses the degree of conviction for other opinions. We assume here the number of members: alternatives and criteria are m, n and k, respectively. Let $p^d_{d'}(k)(d'=1,2,...,m;\ p^d_{d'}(k)\geq 0)$ be the degree of conviction from member d to member d' (including d) with respect to the k-th criterion. However, it is given a positive value by at least one member. So, the following equation holds.

$$\sum_{d'=1}^{m} p^d_{d'}(k) \neq 0 \quad (d=1,2,\ldots,m). \tag{3}$$

Furthermore the sum of $p^d_{d'}(k)$ is normalized in unity.

Let $\mathbf{p}^d(k) = (p^d_1(k), p^d_2(k),..., p^d_{d'}(k),..., p^d_m(k))$ be the vector of degree of conviction of member d, $\mathbf{w}^d = (w^d_1, w^d_2,..., w^d_n)$ be the weights vector of member d, then the vector of the direction of opinion modifying for member d is defined as

$$\mathbf{r}^d(k) = \sum_{d'=1}^{m} p^d_{d'}(k)\cdot \mathbf{w}^{d'} - \mathbf{w}^d \quad (d=1,2,\ldots,m), \tag{4}$$

using the concept of the convinced opinion modifying method. Equation (4) shows the difference of own composite weights vector subtracted from the weighted average of the composite weights of all members. Substituting $\mathbf{r}^d(k)$ of equation (4) for the $\mathbf{r}(k)$ in equation (2), we can calculate the degree of effectiveness of conviction $g^d_{ij}(k)$ for member d. When all $g^d_{ij}(k)$ is obtained, we request a modification of the pairwise comparison value $a^d_{ij}(k)$ of member d. This request is confined to the relevant member, but we do not force a modification of opinion on any other member.

When the modifying is completed, we return to the calculation process for composite weight using the modified pairwise comparison value $a^d_{ij}(k)$.

4. Experimental Example

In this section, we present the result of an experiment using the proposed method. In this experiment, we introduce the method to a group decision-making problem, and then we consider the reaction of members and post-evaluation.

4.1 Outline of Experiment

In this experiment, we have made the hierarchical diagram beforehand. The problem is a choice of dwelling, and the hierarchical structure diagram and the means of the criteria are shown in Figure 4 and Table 1, respectively.

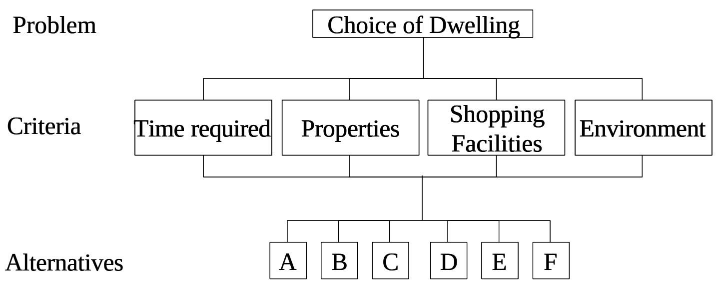


Figure 4. Hierarchical diagram for choice of dwelling

Table 1. Means of criteria

Criterion	**Meaning**
Time required	Time to the company
Properties	Conditions, easy of living
Shopping facilities	Number of shops, offering, accessibility
Environment	Atmosphere around house, safety

The time limit is 60 minutes, and the agreement condition is that the first choice of all members is in agreement.

In this experiment, we have set the common weight from each criterion to each alternative in advance, so we argue only about the importance of criteria (Table 2).

Table 2. Weights from criteria to alternatives

Criterion	Alternative					
	A	B	C	D	E	F
Time required	0.062	0.186	0.093	0.078	0.465	0.116
Properties	0.426	0.172	0.081	0.181	0.047	0.093
Shopping facilities	0.053	0.031	0.169	0.245	0.109	0.393
Environment	0.068	0.042	0.223	0.356	0.089	0.223

4.2 Results

The time required for the experiment was 57 minutes and three opinion modifications were made. Finally In the end, the first choice of all members was in agreement.

The final weights for alternatives are shown in Table 3, with D being the first choice for all members.

Table 3. Final weights of alternatives

Member	Alternative					
	A	B	C	D	E	F
d_1	0.141	0.083	0.159	**0.249**	0.127	0.241
d_2	0.194	0.108	0.149	**0.245**	0.129	0.175
d_3	0.196	0.110	0.137	**0.222**	0.136	0.199
d_4	0.210	0.116	0.140	**0.232**	0.133	0.169

Next, we observe the process of agreement through the difference between the weight for each member and the geometric mean method (Saaty, 1980). Let n be the number of elements, $\mathbf{w}^d = (w_1^d, w_2^d, ..., w_n^d)$ be the weights vector for member d and $\mathbf{w}^{\text{GM}} = (w_1^{\text{GM}}, w_2^{\text{GM}}, ..., w_n^{\text{GM}})$ be the geometric mean vector, then the Euclid distance between is shown in the next equation,

$$\ell_d = \sqrt{\sum_{i=1}^{n} (w_i^d - w_i^{\text{GM}})^2}. \quad (5)$$

Figure 5 shows the amount of ℓ_d that the horizontal axis is the number of times of opinion declaration. We can observe that the values of ℓ_d decrease as the opinion is modified except for d_1. So the weights of d_2, d_3 and d_4 are approaching the geometric mean.

Figure 6 shows the change of weights for each modification and each member. Let n be the number of elements, $\ell^{d(t)}$ be the change of opinion between the t-th time and the (t+1)-th time, and $\mathbf{w}^{d(t)} = (w_1^{d(t)}, w_2^{d(t)}, ..., w_n^{d(t)})$ be the weight vector on the t-th time, then $\ell^{d(t)}$ is defined as the next equation,

$$\ell^{d(t)} = \sqrt{\sum_{i=1}^{n} (w_i^{d(t+1)} - w_i^{d(t)})^2}. \quad (6)$$

The change of opinion of member d_3 was greater than other members. The third modification was especially large. This indicates that member d_3 has greatly compromised in the third round; his weight is far from the others. This is because our method does not capture the amount of modification.

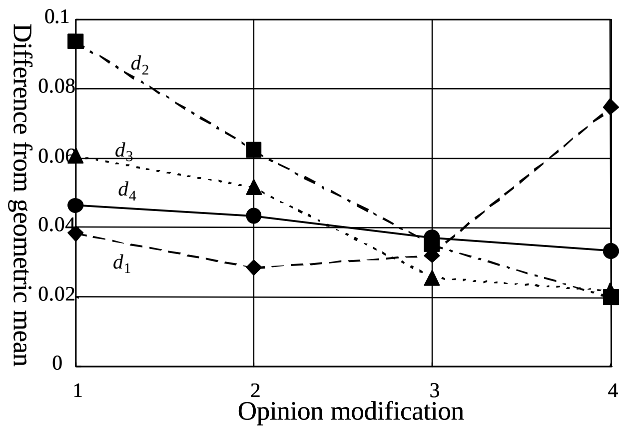


Figure 5. Process of opinion modification

Table3. First choice alternatives each round

Member	1	2	3	4
D_1	A	A	D	D
d_2	D	D	D	D
d_3	A	A	A	D
d_4	A	A	A	D
GM	A	A	D	D

Table 3 shows the first choice each time, GM on the table being the first choice geometric mean. From this table, we see that the agreement is not made linearly. In the first and second round, three members and the GM gave first place to A. Thereafter, it moved gradually to D. If we conclude the first round by using the geometric mean, A is the best alternative. But through discussion and modification of opinion, D was chosen.

The following outlines the major findings from this experiment.

- Most of the members indicated that this method facilitated the modifications of opinion. It is difficult to understand the difference between own and other opinions only through discussions, but the weights that express a quantitative measure make it easier to grasp the position of opinions.
- Some members felt that their opinions were reflected in the final conclusion through repeated discussions and opinion modification. Needless to say, this is an objective of group decision-

making. When a member feels that his/her opinion is reflected usefully in the decision-making, then dissatisfaction may decrease even if his/her suggestion is far from the final conclusion. On the other hand, a member who has changed his/her opinion substantially may be dissatisfied with the final conclusion. This kind of comment may have emerged because our method allows members to grasp quantitatively the difference between his/her opinion and the final conclusion.

- Many members pointed out that the method was too time-consuming. Some procedures, e.g. weight calculation, have been automated, but our method includes many repetitive processes, i.e. discussion, presentation of weight, weight calculation and feedback of results, so some members' feelings were somewhat ambivalent.

5. Concluding Remarks

In this paper, we proposed a method fro group decision-making using AHP. Our method focused on the discussion process and provided the information for modifying members' opinions. In an experiment of group decision-making, most members accepted our method, because it did not ignore all the opinions. Furthermore, it was possible to grasp the change in members' opinions and the degree of compromise by observing the moving weights in the opinion modifying process.

Even that, many operations are not automated; a computer system to facilitate the proposed method is required. Moreover, we dealt only with AHP. In our future research, we will extend our method to ANP and other mutual evaluation methods.

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