DERIVING GROUP PRIORITIES IN THE FUZZY ANALYTIC HIERARCHY PROCESS

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Summary: *A new Group Fuzzy Preference Programming (GFPP) method for fuzzy prioritisation in the Analytic Hierarchy Process is proposed. The assessment of the priorities from fuzzy pairwise comparison judgements is formulated as an optimisation problem, maximising the group's overall satisfaction with the group solution. Unlike the existing fuzzy prioritisation methods, the GFPP method derives crisp priority vectors and does not need additional fuzzy ranking procedures. The proposed method can easily deal with missing expert's judgements and provides a meaningful indicator for measuring the level of group satisfaction and group consistency.*

1. Introduction

Group decision-making is considered as a process of deriving a single group preference from a number of individual preferences regarding a finite set of criteria and alternatives. In the AHP, which is one of the most popular techniques for individual and group decision-making, the individual preferences are represented as comparison judgements and their intensity is measured on a ratio scale [12].

The pairwise comparison in the AHP assumes that the decision maker can compare any two elements E_i, E_j at the same level of the hierarchy and to provide a positive numerical value a_{ij} of the ratio of their importance. If the element E_i is preferred to E_j then $a_{ij} > 1$. Correspondingly, the reciprocal property $a_{ji} = 1/a_{ij}$ always holds.

For a level with n elements a full set of $n(n-1)/2$ judgements is needed in order to construct a reciprocal matrix of pairwise comparisons $A = \{a_{ij}\} \in \Re^{nxn}$.

The prioritisation process in the AHP consists in deriving a priority vector $w = (w_1, w_2, ..., w_n)^T$ from each comparison matrix $A = \{a_{ij}\}$. The set of n relative priorities should be normalised to sum of one, $\sum_{i=1}^{n} w_i = 1$, $w_i > 0$, $i = 1,2,...,n$, so the number of independent priorities is *(n-1)*. Traditionally the prioritisation process is based on the Eigenvalue or Logarithmic least squares methods [11], although there are some other appropriate methods, briefly reviewed in [9].

The group prioritisation process in the AHP requires construction of pairwise comparison matrices at each level of hierarchy either by consensus voting or by aggregating the individual preferences [1], [2], [12]. In the consensus voting all group members agree upon the values for each comparison judgement. If the group is unwilling or unable to vote or cannot achieve a consensus, then a compromise group solution can be obtained by combining the individual preferences into aggregated group preferences.

However, in many cases the preference models of the human decision-makers are uncertain and fuzzy and it is relatively difficult crisp numerical values of the comparison ratios to be provided. The decision-makers may be uncertain about their level of preference due to incomplete information or knowledge, inherent complexity and uncertainty within the decision environment, lack of an appropriate measure or scale. Such problems are very likely to occur in the group decision-making process, as the group members frequently have different levels of expertise. Therefore an appropriate group prioritisation method should be able to deal with situations, where there are uncertain judgements as well.

A natural way to cope with the uncertain judgements is to express the comparison ratios as fuzzy sets or fuzzy numbers, which incorporate the vagueness of the human thinking. When comparing two elements E_i and E_j, the exact numerical ratio a_{ij} could be approximated with a fuzzy ratio "about a_{ij}", which is represented by a fuzzy number $\tilde{a}_{ij}$.

Van Laarhoven and Pedrycz [13] extend the AHP to fuzzy hierarchical analysis, using comparison matrices with triangular fuzzy numbers. They derive fuzzy group prioritity vector $\tilde{w}$ by minimisation of a logarithmic regression function. Because of the dependency between the resulting equations, they apply a normalisation procedure to find a unique solution. But, as discussed by the authors, in some cases their approach may result in irrational fuzzy priorities, where the normalised upper value < normalised mean value < normalised lower value.

Boender et al. [4] show that the normalisation procedure used by van Laarhoven and Pedrycz yields a loss of optimality, and propose a modified normalisation procedure. However, as it is shown in [8], their method might also produce irrational fuzzy priorities.

Buckley [6] employs fuzzy trapezoidal numbers, claiming that they are more easily understood and used by the experts. The individual fuzzy reciprocal matrices, provided by each group member for a specific level of the hierarchy are aggregated into a single 'group' matrix by using a geometric mean. The group prioritisation process consists in finding fuzzy group priorities from aggregated fuzzy matrices and is based on a fuzzy modification of the Logarithmic least squares method. The derived fuzzy priorities are then combined in the Saaty hierarchy to compute the final fuzzy scores of alternatives, which are compared by fuzzy ranking.

The Buckley's method always derives rational priorities, but it needs a full set of fuzzy judgements for construction of fuzzy reciprocal matrices. The construction of fuzzy reciprocal matrices, taken by analogy from the crisp prioritisation methods however leads to some inaccuracy due to the non-linearity of the Saaty scale. Moreover, the skewed reciprocals might reverse the final ranking of the elements if an inverse ratio scale is used, as shown in [10].

Another approach for fuzzy group prioritisation, called Synthetic extent analysis, is given in [7]. The author initially aggregates the individual fuzzy comparison matrices into a group fuzzy matrix and applies a simple arithmetic mean algorithm to find fuzzy priorities from the group matrix. However, the arithmetic mean is a very naïve prioritisation approach, as shown by Saaty [11], and is appropriate only if the comparison matrices are consistent, which is very unlikely in the group decision process.

A common feature of all existing fuzzy prioritisation methods is that they derive *fuzzy* priorities, and after aggregating, the final scores of the alternatives are also represented as fuzzy numbers or fuzzy sets. Due to the large number of multiplication and addition operations, the resulting fuzzy scores have wide supports and overlap over a large range.

These methods require an additional *fuzzy ranking procedure* to find the final ranking of the alternatives from the fuzzy scores. The different ranking procedures, however, often give different ranking results [5].

Most of the known methods derive priorities from fuzzy comparison *matrices*. Despite the problems with the skewed reciprocals, already mentioned, in the group decision-making process some group members might be unwilling or unable to provide all fuzzy comparisons necessary to construct full comparison matrices.

In this paper, a new Group Fuzzy Preference Programming (GFPP) method for deriving group priorities from fuzzy comparison judgements is proposed, which eliminates some of the drawbacks of the existing fuzzy prioritisation methods. The assessment of the priorities from a set of fuzzy pairwise comparison judgements is formulated as an optimisation problem, maximising the group's overall satisfaction with the group solution.

Unlike the existing fuzzy prioritisation methods, the GFPP derives *crisp* priority vectors, which eliminates the need of an additional fuzzy ranking procedure and fuzzy normalisation. The proposed method does not require construction of fuzzy comparison matrices and can easily deal with missing expert's judgements. The method also provides a meaningful indicator for measuring the level of the group consistency, which does not exist in the known fuzzy prioritisation methods.

2. Group Fuzzy Preference Programming Method

The group fuzzy prioritisation problem in the AHP can be formulated in the following way: Consider a group of *K* experts, comparing pairwisely *n* elements at the same level of hierarchy. Each expert provides a set of fuzzy comparison judgements $A_k = \{\tilde{a}_{ijk} \mid i = 1,2,\ldots,n-1,\ j = 2,3,\ldots,n;\ j > i\}$, $k = 1,2,\ldots,K$, where $\tilde{a}_{ijk}$ represents the relative importance of the decision element E_i over E_j, with respect to an upper level element, assessed by the *k*-th expert. The fuzzy judgements are represented as triangular fuzzy numbers $\tilde{a}_{ijk} = (l_{ijk}, m_{ijk}, u_{ijk})$, where l_{ijk}, m_{ijk} and u_{ijk} are their lower, modal and upper bounds correspondingly. The problem is to derive a crisp group priority vector $w = (w_1, w_2, \ldots, w_n)^T$, such that the priority ratios w_i / w_j approximately satisfy the initial fuzzy judgements:

$$l_{ijk} \tilde{\leq} \frac{w_i}{w_j} \tilde{\leq} u_{ijk}, \qquad (1)$$

$$i = 1,2,\ldots,n-1,\ j = 2,3,\ldots,n,\ j > i,\ k = 1,2,\ldots,K,$$

where the symbol $\tilde{\leq}$ denotes 'fuzzy less or equal to', or in linguistic terms, 'approximately less or equal to'.

A Fuzzy Preference Programming (FPP) method was previously proposed by the author of this paper [9] for deriving priority vectors from a set of crisp comparison judgements of a single decision-maker. This method represents the prioritisation process as an optimisation problem for obtaining a priority vector, maximising the decision-maker's overall satisfaction with the final solution.

Two modifications of the FPP method are given in [10] for obtaining priorities from fuzzy comparison judgements of a single decision-maker. The first one is based on a-cuts decompositions and transforms the prioritisation problem into a linear program. The second modification formulates the fuzzy prioritisation problem as an optimisation task, similar to the previous one, but it requires the solution of a non-linear program. The non-linear modification of the FPP method derives crisp priorities and has the advantage that it does not need additional aggregation and ranking procedures. It can easily be applied for solving the above group decision-making problem, as it is shown below.

For a given priority vector $w = (w_1, .., w_n)^T$, the degree of approximation, or satisfaction with each double-side inequality (1) can be measured by a membership function, which is linear with respect to the unknown ratio w_i / w_j:

$$
\mu_{ijk}(w)=\begin{cases}\dfrac{(\frac{w_i}{w_j}-l_{ijk})}{m_{ijk}-l_{ijk}}, & \dfrac{w_i}{w_j}\le m_{ijk}\\[2ex] \dfrac{(u_{ijk}-\frac{w_i}{w_j})}{u_{ijk}-m_{ijk}}, & \dfrac{w_i}{w_j}\ge m_{ijk}\end{cases}. \tag{2}
$$

In order to avoid dividing by zero, we will assume that $u_{ijk} > m_{ijk} > l_{ijk}$. Actually, this is not a binding assumption, since certain judgements can be represented as triangular fuzzy numbers with very small scope $d_{ijk} = (u_{ijk} - l_{ijk})$. Obviously, the scopes of the fuzzy judgements correspond to the degree of uncertainty of the decision-maker with respect to comparison ratios.

The membership function (2) is linearly increasing over the interval $(-\infty, m_{ijk})$ and linearly decreasing over the interval (m_{ijk}, ∞). It takes negative values when $w_i / w_j < l_{ijk}$ or $w_i / w_j > u_{ijk}$ and has a maximum value $\mu_{ijk} = 1$ when $w_i / w_j = m_{ijk}$. Over the range $(l_{ijk},\ u_{ijk})$ the membership function (2) coincides with the fuzzy triangular judgement $\tilde{a}_{ijk} = (l_{ijk},\ m_{ijk},\ u_{ijk})$.

The fact that the functions (2) is not bounded from below allow us a to measure not only the degree of satisfaction, but also the degree of dissatisfaction with some crisp priority vector. The degree of membership $\mu_{ijk}(w)$ is positive for $w_i / w_j \in [l_{ijk}, u_{ijk}]$, when the decision-maker is satisfied with the corresponding priority vector, and $\mu_{ijk}(w) < 0$ for $w_i / w_j \notin [l_{ijk}, u_{ijk}]$, when the decision-maker is completely dissatisfied.

The solution to the prioritisation problem by the FPP method is based on two additional assumptions [9], [10]. The first one requires the existence of nonempty fuzzy feasible areas P_k for each expert on the *(n-1)*-dimensional simplex Q^{n-1},

$$
Q^{n-1} = \left\{ (w_1, \ldots, w_n) \middle| w_i > 0, \sum_{i=1}^{n} w_i = 1 \right\}, \tag{3}
$$

defined as intersection of the membership functions (2), corresponding to the judgements of the *k*-th expert and the simplex (3). The membership function of the *k*-th fuzzy feasible area P_k is given by

$$
\mu_{P_k}(w) = \min_{ij} \{ \mu_{ijk}(w) | i = 1, \ldots, n-1;\ j = 2, \ldots, n;\ j > i \} \tag{4}
$$

By defining the membership functions (2) as L-fuzzy sets $\{L = [-\infty, 1]\}$, we can relax the assumption of non-emptiness of P_k on the simplex. If the fuzzy judgements of the *k*-th expert are very inconsistent, then $\mu_{P_k}(w)$ could take negative values for all normalised priority vectors $w \in Q^{n-1}$.

The second assumption specifies a selection rule, which determines a priority vector, having the highest degree of membership in the aggregated membership function (4). It can easily be proved as in [9] that $\mu_{P_k}(w)$ is a convex set, so there is always a priority vector $w^k = (w_{1k}, \ldots, w_{nk})^T$ in Q^{n-1} that has a maximum degree of membership λ^k:

$$
\lambda^k = \mu_{P_k}(w^k) = \max_{w \in Q^{n-1}} \min_{ij} \{ \mu_{ijk}(w) \}. \tag{5}
$$

It is seen that the prioritisation procedure of the FPP method is based on the maximin decision rule, known from the game theory. The maximin rule has also been applied by Bellman and Zadeh [3] for solving decision-making problems in uncertain environment. Zimmermann [14] uses the same decision rule for fuzzy linear problems with soft constraints and shows, that if the membership functions, representing the soft constraints are linear, the maximin problem can be transformed into a linear programming problem. Similar approach is also applied in [9] to formulate the prioritisation problem as a linear program.

However, for group applications of the FPP method there is no need to solve individual maximin problems of the type (5) and then to aggregate the individual priorities, using additional aggregation procedures, as arithmetic or geometric means. Instead, we can construct a group fuzzy feasible area $P = \bigcap_{k=1}^{K} P_k$ on the simplex Q^{n-1}, and define a group membership function $m_P(w)$ as an intersection of all membership functions (4):

$$m_P(w) = \bigcap_{k=1}^{K} m_{P_k}(w) = \min_{w \in Q^{n-1}} \{ m_{ijk}(w) \big| i = 1,\ldots,n-1;\, j = 2,\ldots,n;\ j > i; k = 1,\ldots,K \} \qquad (6)$$

Then, applying the maximin decision rule we have

$$l = \max_{w \in Q^{n-1}} \min \{ m_{ijk}(w) \big| i = 1,\ldots,n-1;\ j = 2,\ldots,n;\ j > i; k = 1,\ldots,K \} \qquad (7)$$

The maximin group prioritisation problem (7) can be represented in the following way:

maximise λ
subject to
$l \le m_{ijk}(w)$,
$i = 1,2,\ldots,n-1$, $j = 2,3,\ldots,n$, $j > i$, $k = 1,2,\ldots,K$, (8)
$\sum_{l=1}^{n} w_l = 1$, $w_l > 0$, $l = 1,2,\ldots,n$.

Taking into consideration the specific form of the individual membership functions (2), the above problem can be transformed into a bilinear program of the type:

maximise λ
subject to
$(m_{ijk} - l_{ijk})\, l\, w_j - w_i + l_{ijk} w_j \le 0$,
$(u_{ijk} - m_{ijk})\, l\, w_j + w_i - u_{ijk} w_j \le 0$, (9)
$\sum_{l=1}^{n} w_l = 1$, $w_l > 0$, $l = 1,2,\ldots,n$
$i = 1,2,\ldots,n-1$, $j = 2,3,\ldots,n$, $j > i$, $k = 1,2,\ldots,K$.

The assumption that the group membership function (6) is composed as intersection of nonempty convex L-fuzzy sets implies that the program (9) always has a solution (w^*, l^*), such that $w^* \in Q^{n-1}$ and $-\infty < l^* \le 1$. The optimal priority vector w^* has a maximum degree of membership in the group membership function (6), given by the second component of the optimal solution, $l^* = m_p(w^*)$.

If all group judgements $\tilde{a}_{ijk}$ are perfectly consistent (which however is very unlikely in the group decision process), then the value of l^* is equal to one. In this case the solution ratios are equal to the modal values of the fuzzy judgements, $w_i^* / w_j^* = m_{ijk}$.

The optimal value of l^*, if it is positive, indicates that all solution ratios completely satisfy the initial judgments, i.e. $l_{ijk} \le w_i^* / w_j^* \le u_{ijk}$. When the judgements are very inconsistent or the scopes of the fuzzy judgements are very small, the group membership function (6) could become negative, $m_P(w)$ <0 for all $w \in Q^{n-1}$. In that case the optimal solution to (9) corresponds to a negative value of l^*, which indicates the degree of group dissatisfaction. It follows that l^* measures the overall group satisfaction or dissatisfaction with the optimal priority vector w^*, derived by (9) and can be used as a group consistency index.

If some of the experts have greater power, then their judgements are more important and should be satisfied in a higher degree from the group solution. This could be done by introducing additional expert weights that modify the individual membership functions (2).

An advantage of the proposed approach is that it combines the synthesis and integration stages of the group decision process into a coherent framework, so there is no need for an additional aggregation of the individual judgements or individual priorities by arithmetic or geometric means. The group priority vector w^* satisfies all expert judgements in a degree equal or greater than the group consistency index l^*, i.e. it ensures equal minimum degrees of satisfaction of all experts.

The existence of a consistency index is a very attractive feature of the proposed fuzzy prioritisation method, which is illustrated in the next section. It can also be observed, that the non-linear program (9) does not necessarily need a full set of all fuzzy judgements from the upper triangular part of the comparison matrices. Therefore, the proposed method can derive priorities from incomplete set of judgements, which is another appealing feature of our approach.

3. Numerical examples

We will present two examples in order to illustrate the GFPP method, proposed in this paper. The first example shows an optimal solution, satisfying all judgements, whereas the second one illustrates a problem with missing judgements and strong group inconsistency. The second problem is widely discussed in the literature for comparison of different normalisation procedures [4], [8], [13] and results in an irrational fuzzy priority vector when the fuzzy logarithmic least square method is applied.

Example 1

Consider two experts, comparing pairwisely three elements at a given level of hierarchy and providing the following fuzzy judgements:

$\tilde{a}_{121}$ =(1, 2, 4); $\tilde{a}_{131}$ =(3, 5, 6); $\tilde{a}_{231}$ =(2, 3, 4);
$\tilde{a}_{122}$ =(2, 3, 4); $\tilde{a}_{132}$ =(5, 6, 8), $\tilde{a}_{231}$ =(2, 3, 4).

The GFPP solution to the problem $w = (w_1, w_2, w_3)^T$ is given in Table 1. It can be observed that the group priorities satisfy approximately all initial judgements. It is also seen that the group solution ensures degrees of satisfaction with each judgement greater than or equal to the group consistency index l.

Table 1. Group solution (Example 1)

w_1	w_2	w_3	l	$\frac{w_1}{w_2}$	$\frac{w_1}{w_3}$	$\frac{w_2}{w_3}$	m_{121}	m_{131}	m_{231}	m_{122}	m_{132}	m_{232}
.6253	.2636	.1111	.3723	2.372	5.628	2.372	.8139	.3723	.3723	.3723	.6277	.3723

The fuzzy normalised results obtained by aggregating the fuzzy judgements by a geometric mean, constructing a full comparison matrix and applying the logarithmic least square method [6] with a consequent normalisation are given below for comparison:

$\tilde{w}_1$ =(0.356, 0.619, 1.048),
$\tilde{w}_2$ =(0.160, 0.279, 0.489),
$\tilde{w}_3$ =(0.067, 0.102, 0.175).

In order to illustrate the properties of the group GFPP solution, the individual priorities, which are determined by solving (9) with the constrains of a single expert only are shown in Table 2:

Table 2. Individual solutions (Example 1)

k	w_{1k}	w_{2k}	w_{3k}	l_k	$\frac{w_{1k}}{w_{2k}}$	$\frac{w_{1k}}{w_{3k}}$	$\frac{w_{2k}}{w_{3k}}$	m_{12k}	m_{13k}	m_{23k}
1	.5746	.3143	.1111	.828	1.828	5.172	2.828	.8284	.8284	.8284
2	.6531	.2507	.0962	.6061	2.606	6.789	2.605	.6061	.6061	.6061

From Table 2 we can observe that the individual priorities of each expert equally satisfy his initial judgements with a degree, equal to the individual consistency index l_k. The values of the group solutions, Table 1 are in between the individual ones, hence they satisfy the Pareto social choice axiom. It is also seen that the inconsistency of the group solution is greater than the individual inconsistencies, which is not unexpected.

Example 2

Three experts compare three alternatives and provide an incomplete set of fuzzy judgements [4], [8], [13]:

$\tilde{a}_{121}$ =(2.5, 3, 3.5); $\tilde{a}_{131}$ =(2.5, 3, 3.5); $\tilde{a}_{231}$ =(2/3, 1, 3/2);
$\tilde{a}_{122}$ =(2.5, 3, 3.5);
$\tilde{a}_{123}$ =(1.5, 2, 2.5).

The group priority vector to the above problem obtained by the GFPP method is shown in Table 3. It can be observed that the group consistency index l is equal to zero, which indicates that some of the judgements are not completely satisfied. In fact all judgements except $\tilde{a}_{12k}$ are satisfied with degrees greater than zero. The obtained solution ratio w_1 / w_2 has zero degree of membership in all m_{12k}, because there is no overlap between $\tilde{a}_{121} = \tilde{a}_{122}$ =(2.5, 3, 3.5) and $\tilde{a}_{123}$ =(1.5, 2, 2.5), but the ratio w_1 / w_2 is equal to the values of their lower and upper bounds correspondingly, so all judgements $\tilde{a}_{12k}$ are equally satisfied.

Table 3. Group solution (Example 2)

w_1	w_2	w_3	l	$\frac{w_1}{w_2}$	$\frac{w_1}{w_3}$	$\frac{w_2}{w_3}$	m_{121}	m_{131}	m_{231}	m_{122}	m_{123}
.5646	.2259	.2095	0.0	2.5	2.6951	1.078	0.0	.39	.844	0.0	0.0

The fuzzy priorities derived by the Logarithmic least square method by applying the normalisation procedure, proposed by Boender et al. [4], are taken from [8]:

$\tilde{w}_1$ =(0.5825, 0.5789, 0.5597),
$\tilde{w}_2$ =(0.1753, 0.2167, 0.2707),
$\tilde{w}_3$ =(0.1699, 0.2045, 0.2475).

It is seen that the means of these fuzzy priorities are close to our results, but the first priority is an irrational fuzzy number, since its lower bound is greater than the upper one.

4. Conclusions

A new method for group prioritisation in the AHP is proposed, based on a fuzzy optimisation approach, which maximises the group satisfaction with the final group solution. The GFPP method derives crisp priorities from fuzzy comparison judgements and eliminates some of the drawbacks of the existing fuzzy prioritisation methods.

The method combines the synthesis and prioritisation stages into a single integrated stage, without requiring additional aggregation procedures, such as arithmetic or geometric means and provides an appropriate index for measuring the consistency of the group solution.

All these features make the proposed method a suitable alternative of the existing fuzzy methods for group decision-making with the AHP.

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