MIND TRANSITION MODEL
-A UNIFIED MODEL OF AHP AND ANP-

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Summary: *Mind Transition Model (MTM), which is a unified model of AHP and ANP, is proposed. MTM is a Markov chain network representing one's mental state transition. Both AHP and ANP can be interpreted as MTM networks.*

1. Introduction

A unified treatment of AHP and ANP is presented in this paper. We propose MTM, or Mind Transition Model, which is a unified model of AHP and ANP. MTM is a Markov chain network with its states corresponding to one's mind states and its transition arcs corresponding to one's mind transitions. MTM enables one simulate one's mind transition profile, thus with MTM we can simulate the mind transition profiles when using AHP and ANP. Both in AHP and ANP the priority weight of an element corresponds to the stationary state probability of the element. The bigger is the stationary state probability of an element, the higher is the priority weight of the element. The priority weight of an element is interpreted being proportional to the time duration of the mind staying in the state for the element.

In Chapter 2, AHP and ANP are briefly explained, and in Chapter 3, MTM is presented. In Chapter 4, AHP is interpreted as an MTM. In Chapter 5, two ANP networks, strongly-connected bipartite ANP and nonstrongly-connected three-level ANP, are interpreted as MTMs. In Chapter 6, a numerical example is presented for the nonstrongly-connected three-level ANP.

2.AHP and ANP

AHP consists of the three steps. The first step is decomposition, or structuring the problem into a hierarchy. The second step is comparative judgment, or eliciting judgments about the relative importance of the elements and estimating priority weight of each element. The third step is path synthesis, or multiplying each local priority weight of elements along a path and summing up them over all the paths between the goal and an alternative to produce the global priority weight of the alternative.

ANP, which is a generalization of AHP, also consists of three steps. The first step is decomposition, which is the same as in AHP except that the problem is structured into a network, instead of a hierarchy. The second step is comparative judgment, which is quite the same as in AHP. The third step is path synthesis, but here path weight need be synthesized into the global priority weight in an ergodic manner. The difference and similarities of AHP and ANP are summarized in Table1

Table 1 Comparison of AHP and ANP

	AHP	ANP
Structure	Hierarchy	Network
Local priority weight determination	Paired comparison,etc	Paired comparison,etc
Alternative priority weight determination	Path synthesis method,etc	Power method,etc
Markov chain	Non-ergodic(or transient)	Ergodic and non-ergodic

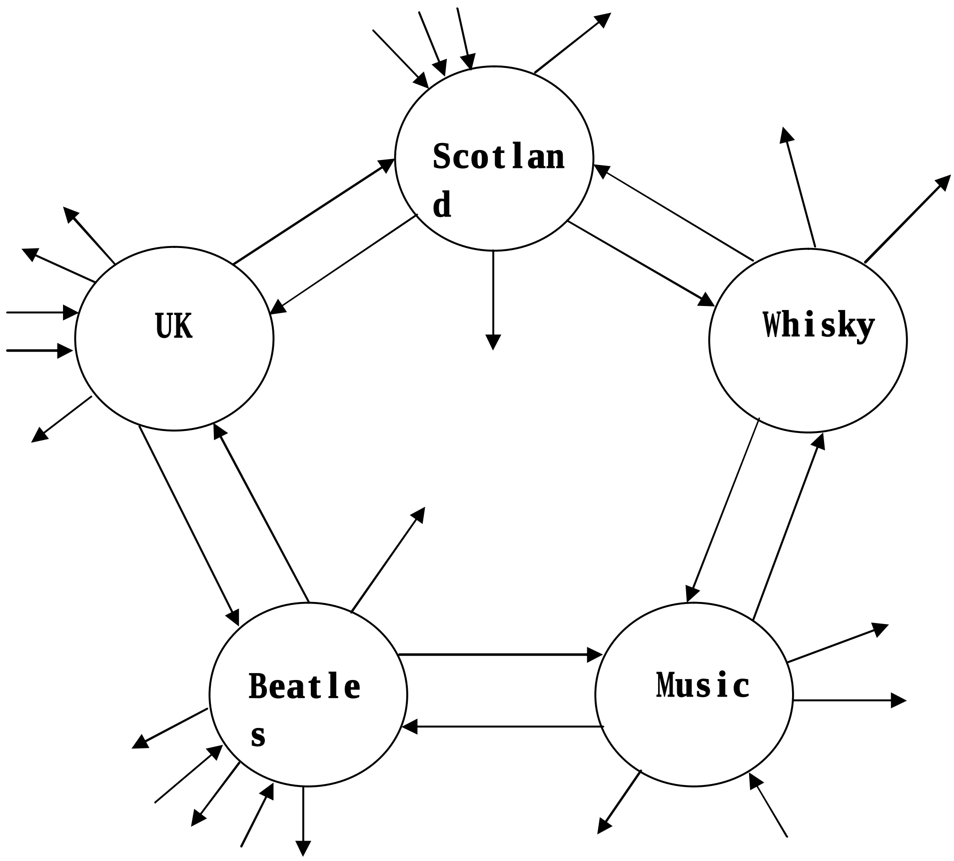


Fig.1 Example of mind transition network

3. Mind Transition Model (MTM)

Mind Transition Model, or MTM, is a Markov chain network whose nodes correspond to mind states and arcs correspond to mind transitions. When one thinks of something, we can say that his/her mind is in the state of the something. At the next moment, he/she may think of another thing. Then, his/her mind state may change to the other thing. Figure 1 shows the mind transition network when I think of "Scotland", where IFORS2002 was held. At the next moment I may think of "UK" or "Whisky". When my mind is in the state of "Whisky", I may be feeling relaxed and think of "Music". Thinking of "Music", I think of "Beatles". Thinking of "Beatles", I think of "UK" again, and so on.

How long one's mind stays at a state or at a group of states reflects how frequently he/she thinks of the element associated with of the state or the concept associated with the group of states. So the stationary state probability of the state for an element can be, in a sense, interpreted as the priority weight of the element. It is questionable whether or not the priority weight is linearly proportional to the stationary state probability. But we assume they are linear proportional to each other. Next in Chapter 4 we can show that the priority weight of an alternative in AHP network is equal to the stationary state probability of the alternative, being normalized so that the sum of the stationary state probabilities over the whole alternatives is unity.

4. AHP as MTM

We consider AHP version of Saaty's car-buying example to illustrate how an AHP network can be converted into an MTM network. Figure 2 shows the AHP network for this car-buying example. Priority weights estimated in Tables 2.1, 2.2 and 2.3 are marked on outgoing arcs from each criterion node. W=(w_1, w_2, w_3) is the criterion choice priority vector, and they should satisfy that $w_1 + w_2 + w_3 = 1$. A method of transforming an AHP network into an ergodic Markov chain MTM network will be presented next. As explained in Chapter 2, an AHP network can be interpreted as a transient, or non-ergodic, Markov chain network. Since the purpose of this paper is proposal of a unified model of AHP and ANP on the basis of Markov chain MTM network, we try to transform an originally non-ergodic AHP network into an ergodic Markov chain MTM network. One transformation is illustrated in Figure 3, where an arc to the goal node with transition probability=1 is added to each of the three alternative nodes. Since priority weights all sum up to unity at each criterion node, these priority weights can be regarded as Markov chain transition probabilities. So the resultant network is strongly-connected, and is an ergodic Markov chain with period=3. For an ergodic Markov chain with period=3, following Theorem 1 holds.

Theorem 1

Let N_i be the node set corresponding to the ith layer of an ergodic Markov chain with period 3, then it holds that $\sum_{j \in N_i} x_j = \frac{1}{3}$.

Here, x_j is the stationary state probability for the state j. ?

For the application to an AHP with more than three levels, this theorem can be generalized as follows.

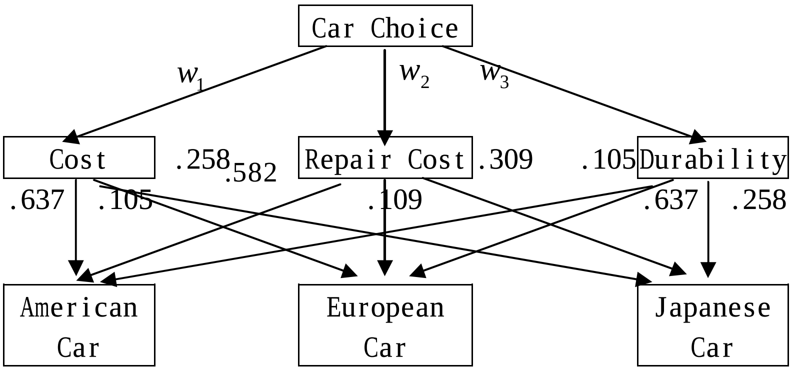


Fig.2 AHP network for car-buying example

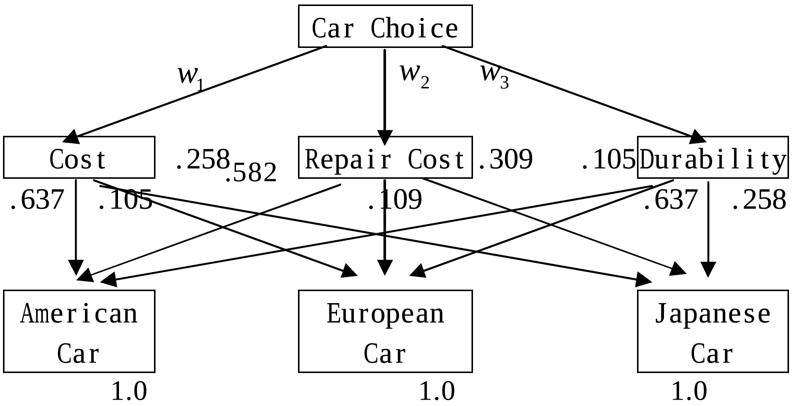


Fig.3 Ergodic MTM network for car-buying example

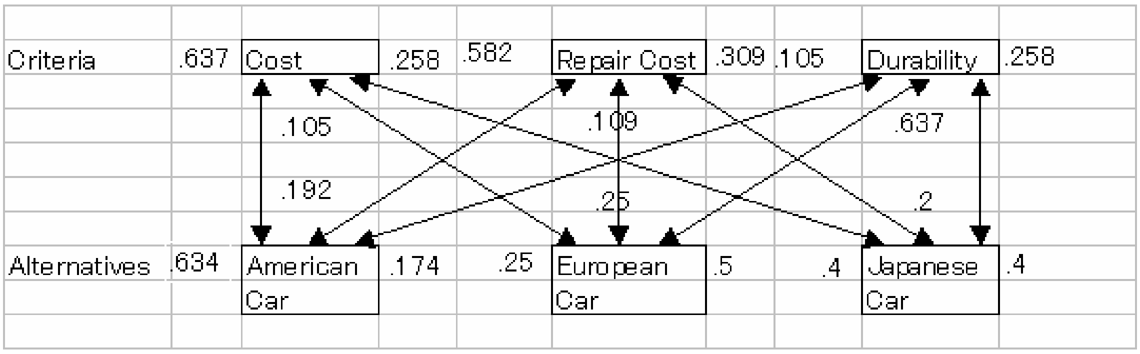


Fig.4 Strongly-connected bipartite ANP for car choice example

Theorem 2

Let N_i be the node set corresponding to the ith layer of an ergodic Markov chain with period=K, then it holds that $\sum_{j \in N_i} x_j = \frac{1}{K}$. ?

Example 1

The transition probability matrix P for the ergodic Markov chain in Figure 3 is given by Eq.(1).

$$
P = \begin{array}{c|ccccccc|}
 & G & C_1 & C_2 & C_3 & A_1 & A_2 & A_3 \\
\hline
G & 0 & w_1 & w_2 & w_3 & 0 & 0 & 0 \\
C_1 & 0 & 0 & 0 & 0 & .635 & .105 & .258 \\
C_2 & 0 & 0 & 0 & 0 & .582 & .109 & .309 \\
C_3 & 0 & 0 & 0 & 0 & .105 & .637 & .258 \\
A_1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
A_2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
A_3 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{array} \qquad (1)
$$

Solving the stationary equilibrium equation $\boldsymbol{x}=\boldsymbol{x}P$ (2) ($\sum x_j = 1$ (3)) for the case of w_1=0.1, w_2=0.2, and w_3=0.7, stationary state probability vector $\mathbf{x}=(x_1, x_2, x_3)$ is obtained. Here, $x_1 = 0.333$, x_2=(0.033, 0.067, 0.233), x_3=(0.085, 0.159, 0.089)

Following the path synthesis formula of AHP, $\boldsymbol{v}=\boldsymbol{w}Q$ (4), the alternative weight vector for the alternatives **v** is calculated.

$\boldsymbol{v}$=(0.254, 0.478, 0.268) (5)

Here, **w**=(0.1, 0.2, 0.7) (6)

$$
Q = \begin{pmatrix} .635 & .105 & .258 \\ .582 & .109 & .309 \\ .105 & .637 & .258 \end{pmatrix} \qquad (7)
$$

Note that it holds that $3x_3=\boldsymbol{v}$, as claimed in Theorem 1. ?

5. Strongly-connected Bipartite ANP interpreted as MTM

We consider a strongly-connected bipartite ANP, or Saaty's car-buying example with outer dependence. In addition to the paired comparison matrices viewed from the criterion, paired comparison matrices viewed from the alternative, are used to form a strongly-connected bipartite ANP of Figure 4. Since the priority weights assigned to outgoing arcs all sum up to unity exactly not only at a criterion node but also at an alternative node, these weights can be regarded as transition probabilities of stochastic matrix (or Markov chain). Therefore, the strongly-connected bipartite ANP is, without any change, an ergodic Markov chain with period=2.

Example 2

The transition probability matrix P for the strongly-connected bipartite ANP of Figure 4 is given by Eq.(8).

Criteria Alternatives

c_1

c_2

c_3

A_1

A_2

A_3

$$P=\begin{bmatrix} 0 & 0 & 0 & 0.637 & 0.105 & 0.259 \\ 0 & 0 & 0 & 0.582 & 0.109 & 0.309 \\ 0 & 0 & 0 & 0.105 & 0.637 & 0.258 \\ 0.634 & 0.192 & 0.174 & 0 & 0 & 0 \\ 0.250 & 0.250 & 0.500 & 0 & 0 & 0 \\ 0.200 & 0.200 & 0.400 & 0 & 0 & 0 \end{bmatrix} \quad (8)$$

$$=\begin{bmatrix} O & P_{12} \\ P_{21} & O \end{bmatrix}$$

Here, O is a zero matrix of appropriate size.

Then, its stationary state probability vector $x=(x_1, x_2)$ is given by Eq.(9) by solving $\boldsymbol{x}=\boldsymbol{x}P$ and $\sum x_j = 1$.

$$\boldsymbol{x}=(x_1,\ x_2) \quad (9)$$

$$x_1 = (0.232, 0.105, 0.163), \quad x_2 = (0.226, 0.1395, 0.1345)$$

From Theorem 2, the priority weight vector for the alternative $\boldsymbol{v}$ is calculated by Eq.(10).

$$\mathbf{v}=2x_2$$
$$=(0.452, 0.279, 0.269) \quad (10)$$

6. Nonstrongly-connected 3-level ANP interpreted as MTM

We consider a nonstrongly-connected 3-level ANP, or Saaty's car-buying example with outer dependence and goal level, which is shown in Figure 5. The 3-level ANP of Figure 5 is obtained either by adding to the bipartite ANP of Figure 4 the goal "Car Choice" and priority weight arcs from the goal to each of the criteria or by adding to the AHP of Figure 3 priority weight arcs from each of the alternatives to each of the criteria. Since there is no priority weight arcs from the criterion to the goal, the 3-level ANP network is not strongly-connected, and hence the corresponding MTM as it is, is non-ergodic and the goal is a transient state with its stationary state probability being zero. Therefore, we need to modify the original 3-level non-ergodic ANP network to form an ergodic MTM network. Among the many ways of transforming non-ergodic networks into ergodic networks, a method of adding the absent-mind state will be presented next.

As shown in Figure 6, the ergodic Markov chain MTM is constructed by adding a new state, called "absent-mind state", to the original 3-level non-ergodic ANP network. An arc with transition probability p is assigned to the absent-mind state from each of criterion states and alternative states. This probability p is call volatility rate.
Remaining transition probabilities are normalized so as to satisfy the probability condition, or that the sum of transition probabilities from a state is unity. From the

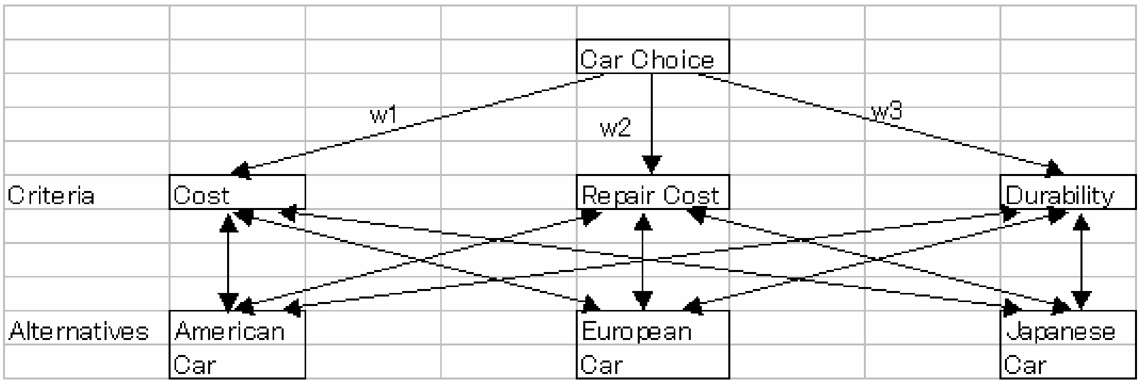
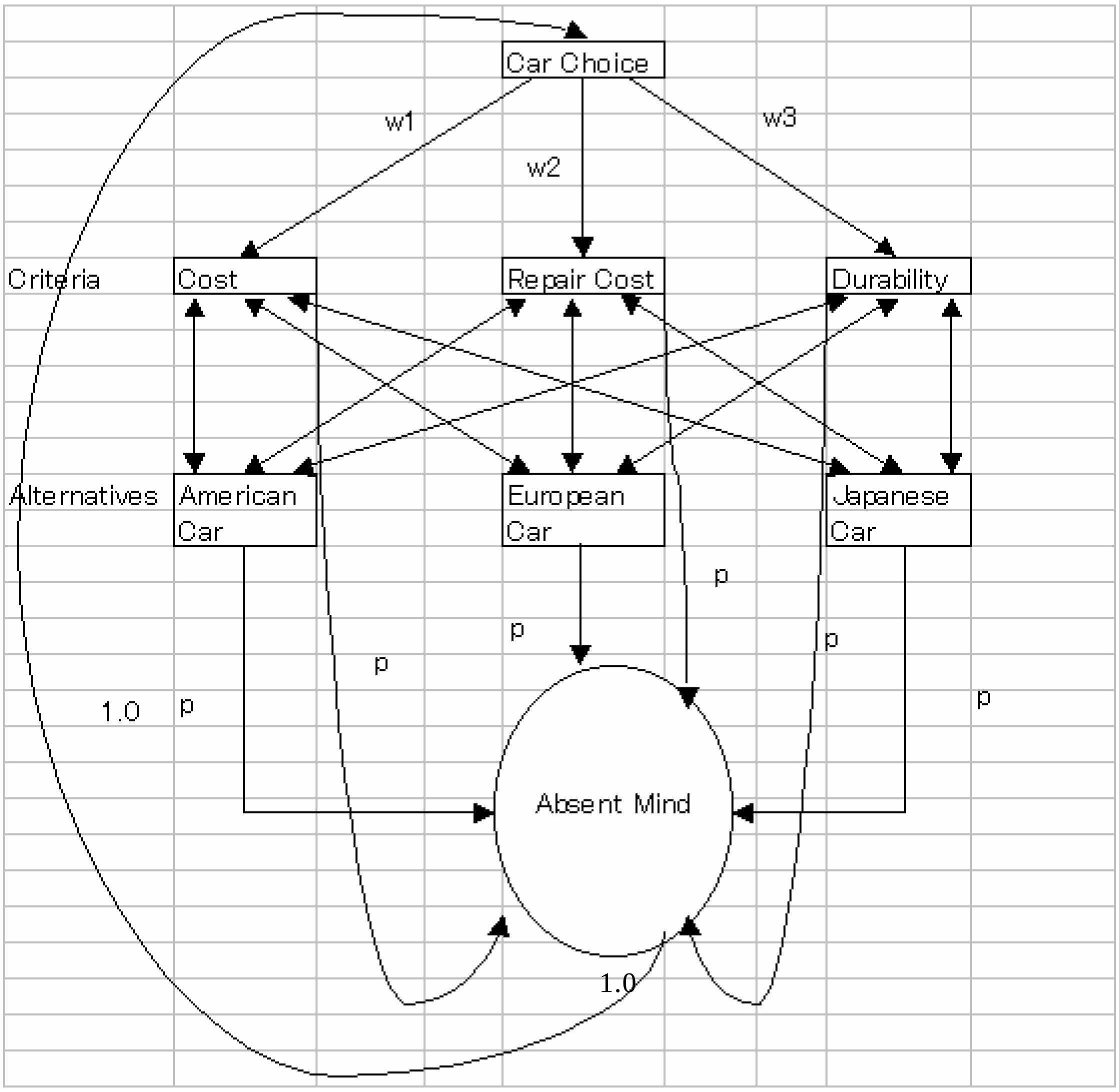


Fig.5 Nonstrongly-connected 3-level ANP for car buying example



absent-mind state to the goal state there assigned an arc with transition probability=1. The resultant Markov chain is ergodic, and moreover, regular (not periodic).

While one is thinking over the car-choice problem, at some time one may think of a criterion and at next moment one may think of an alternative. But with certain probability, or volatility rate p, one may stop thinking over the car-choice problem and think of another thing. This transition out of the present-interesting is modeled by the transition to the absent-mind state. After a while staying in the absent-mind state, one may again start thinking over the car-choice problem, which is represented by the transition from the absent-mind state to the goal state.

Example 3

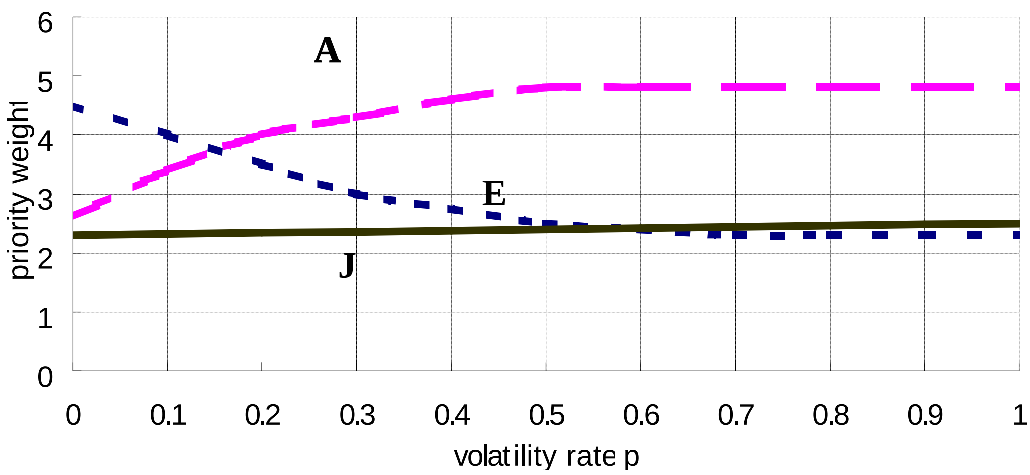
Since the transition matrix P for the bipartite ANP of Figure 4 is given by Eq.(8), the transition matrix M for the ergodic MTM of Figure 6 is given by Eq. (11).

$$M=\begin{bmatrix} 0 & w & 0 & 0 \\ 0 & 0 & (1-p)P_{12} & p1 \\ 0 & (1-p)P_{21} & 0 & p1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \quad (11)$$

$$\boldsymbol{w}=(w_1, w_2, w_3) \quad (12)$$

Here, *1* is a column vector of all 1's and O is a zero matrix of appropriate size.

The stationary state probability vector $x=(x_1, x_2, x_3, x_4)$ is obtained by solving $x=xM$ and $\sum x_j = 1$. Here x_3 is the stationary state probability vector, for the alternatives. Then the priority weight vector for the alternatives is obtained by normalizing x_3 so that the sum of x_3's elements is unity. Figure 7 shows the priority weights of American Car, European Car, and Japanese Car, when the volatility rate p changes from 0 to 1 (w=(0.1, 0.2, 0.7) and w=(0.2, 0.3, 0.5)).



(a) case of w=(0.1,0.2,0.7)

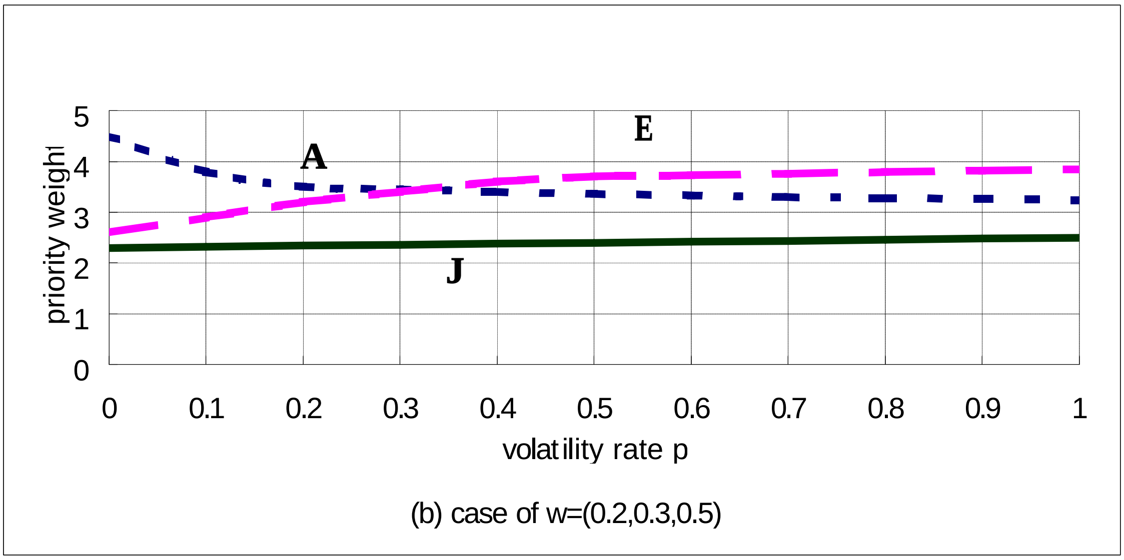


Fig.7 Characteristics of priority weights of three cars with volatility rate p

7.Conclusion

A unified model of AHP and ANP, which is called MTM (Mind Transition Model), is presented in this paper. MTM is a Markov chain network modeling one's mind transition. There can be various types of MTMs for AHP and ANP networks. Other types of MTMs also need be studied, which is a future research subject.

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