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DECISION MAKING BY METHOD OF KEY SUCCESS FACTORS DISCRIMINATION: KEYANP

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Abstract Discrimination of key success factors, critical paths and crucial managerial solutions is widespread in decision making theory and practice. Analytic network process (ANP) is a theory of relative measurements of interconnected alternatives and criteria. This paper offers modified method which considers only the most impostant net elements – KeyANP method. It gives alternative to the classical result. The idea is developed and complete with case study.

Keywords: *KeyANP, decision support system, key success factors discrimination, ANP max/prod*

1. Introduction

Purpose of this paper is consideration and extension of the network analysis models family for multiple criteria decision support system.

Second section of this study is dedicated to Analytic Network Process (ANP). ANP is a theory of network elements absolute priorities determination when priorities of the network elements are established using pairwise comparisons on the basis of expert judgments expressed numerically on an absolute fundamental scale. Analysis discloses that intensional meaning of computations in this method can be characterized by operations **sum/prod** which take into account all possible priority relations in network decision making model.

In Section 3 proposed by us modified method is described. It supposes consideration of only key relations, registration of crucial elements interactions in the network model of decision making in terms of relations **max/prod**. Discrimination of the key management decisions, critical paths, key success factors is a widespread approach in decision making practice. Decision maker may be interested in exactly such a kind of conclusion which results from the analysis of the most important, particularly weighty for the given problem factors and connections. In this section we present formal procedures of elements importance coefficients determination in such a network where extracts a maximal chain of key success factors.

In conclusion, in Section 4, we present a decision making system which allows study participants to look at the analysis object from two different points of view. The first is a classical one, with ANP method basis, second is built upon key factors descrimination in the preference network. A case study of multicriterion analysis which is performed on both of these models is examined. Value of the suggested approach lie in the possibility of problem treatment with the help of system-organized procedure that proposes virious interpritations of decision making model. It enables to make virious conclusions about decision taken.

2. Method of summarized relations consideration in Analytic Network Process: ANP sum/prod

In Analytic Hierarchy Process (AHP) hierarchical structure of criteria and alternatives is used. This traditional method is not able to deal with feadbacks when choosing alternatives. Hierarchy suppose unidirectional top-to-bottom process without cyclic return along decision making chain. AHP method gives general framework of decision taking subject to assumption that elements of the higher level are independent of the lower level elements and that elements within one level have no dependence either.

ANP was proposed by T.Saaty in 1996 and began a new essential phase in decision making theory. In ANP different relations between network model objects are allowed. De facto, ANP uses network without levels specification necessity.

Important contribution of ANP is on that it makes our decisions to bear practical character, because it bases on our causal understanding concerning influence of elements on each other.

Method ANP can be represented as a following srquence of operations:

1. The first stage consists in definition of goals, criteria, subcriteria and alternatives which control system interaction.
2. On the second stage construct network of influences between elements and their groups (clasters). The carriers of interaction are energy, capital, political impact, people.
3. Matrix computations of influence transfer between objects are cariried out by means of special matrix operations on the third stage. Influence spreading repeat cyclically, proceeding to the limit steady-state values. Finally, synthesize the limiting weighed priorities for all alternatives and control criteria.

Graph theory proposes an elegant matrix method of graphs (networks) paths properties exploration. Therefore input data in ANP method presenting by matrix of coupling (of preferences, multiplicative pairwise comparisons) between neighboring graph nodes.

Interactions and mutual influences multiplication effect in ANP is reproduced by numerical method. Therefore one of the main ANP alhorithm operation is an exponentation of pair comparisons matrix $A=|a_{ij}|$, where elements $a_{ij}<1$, and $\sum_j a_{ij}=1$, where i – column, j – row of matrix, $i,j=\overline{1,n}$.

Matrix $A=|a_{ij}|$ is called row-normalized (or stochastic), if for all pairs of elements i,j $a_{ij}\geq 0$, and for all i $\sum_j a_{ij}=1$.

It is established that multiplication of normalized matrices gives also row-normalized matrix.

Example.

$$\begin{pmatrix} 0.1 & 0.4 \\ 0.9 & 0.6 \end{pmatrix}^2 = \begin{pmatrix} 0.37 & 0.28 \\ 0.63 & 0.72 \end{pmatrix}.$$

It is proved that all eigenvalues of normalized matrix don't exceed one, while the sum of its eigenvalues equals one.

If initial matrix A describes one-step relations, then A^2 is a matrix of paths or a matrix of mutual influences with 2-steps length, and A^n is a matrix of path influences with n-steps length. Thus influences spread takes into consideration step by step, cycle after cycle. In this lies accordance between feedback and multipliative effect in preferenes network.

When investigating the process of limit matrix obtaining $A_{lim} = A^n$, where $n\to\infty$, Saaty underlined that multiplication process will not converage if the resulting matrix will be not stochastic after multiplication. But if the initial matrix is stochastic then obtained by means of multiplication matrix is also stochastic. Since all eigenvalues of normalized matrix A don't exceed one, sequence A^n converage to the limiting value A_{lim}. All columns in the limiting matrix blocks will be identical and contain groups of the elements of sought absolute priorities vector.

For any quadratic matrix A product A*A or a matrix A square can be found: $A^2=A*A=|a^2_{i,j}|$. Product $A^k=A*A^{k-1}$ for any integer k is matrix A in the power k. Element $a^2_{i,j}$ of the matrix A^2 calculates as scalar product of the row vector i and the column vector j, i.e. $a^2_{i,j}$ can be found as a sum of pair products of row i items and corresponding items of column j:

$$a^2_{i,j} = a_{i,1} * a_{1,i} + a_{i,2} * a_{2,j} + \ldots + a_{i,l} * a_{l,j} + a_{i,n} * a_{n,j} = \sum_{l=1}^{n} (a_{i,l} * a_{l,j}) \qquad (1)$$

Lets perform a substantial analysis of the operation (1).

Element $a_{i,j}^2$ sums all possible multiplicative pair preference relations between two steps long elements i and j. ***Ordering on basis of the operation (1) corresponds to consideration of all relations and influence factors two steps long*** which are assigned by the matrix of coupling A.

Similarly, matrix A^k in power k contains all paths k steps long between the nodes and relative priority $a_{i,j}^k$ is seeking by means of pair priorities summation along all possible k steps long paths between i and j.

Such method of multi-step pairwise comparisons can be characterized as **sum/prod** method, in accordance with operations in computations by scheme (1)

Calculation of the matrix A^k elements in the form of pair products is used, for example, in the mathematical models of economics for explanation of the meaning of "full costs" as costs which take into account all production cycles when calculating an interindustry balance.

3. Method of the key relations in Analytic Network Process ANP-max/prod

In the decision making theory and practice decision methods which are based on selection of critical paths, key success factors, crucial managerial decisions etc. are widely used.

The information which takes into consideration key relations, key influence factors in the network decision making model can be very useful for decision maker who relies on data from the preference relations matrix A. For that it is better to reject a summation of all mutual influences as it is done in the calculation scheme by formula (1) and to discriminate maximal paths, maximal path relations in analytic network of decision making.

Such information can be obtained if to select a maximal element from the pair products set $\{(a_{i,l} * a_{l,j})\}$ when computing two elements long path relations. And instead of the operation (1) to use operation (2):

$$b_{i,j}^2 = MAX_{l=1}^{n}\{(a_{i,l} * a_{l,j})\} \tag{2}$$

We specify symbol ∘ for the matrix operation (2). Then:

$B^2 = A \circ A = MAX(A*A) = | b_{i,j}^2 |$.

This multicriterion decision making scheme can be designated as **ANP-max/prod** in accordance with operations which have to be carried out when computing path prioroties.

Search process of the key (critical) relations matrices has a cycling character. Thus,

$B^4 = B^2 \circ B^2 = MAX(B^2 * B^2) = | b_{i,j}^4 |$,

$B^{2k} = B^k \circ B^k = MAX(B^k * B^k) = | b_{i,j}^{2k} |$.

In the classical Saaty's sum/prod method initial matrix A has to be normalized, that's why all degrees of this matrix are also normalized. In the computation scheme with key influence factors selection sums of the elements in columns of B^k become less then one:

$\sum_{j=1}^{n} b_{i,j}^k \le 1, \ \forall i$.

To avoid sign loss in the finite processor address space and to save useful for analysis scale of variables representation, it is proposed to normalize matrices of maximal multiplicative path B^k in method **ANP-max/prod** on every iteration. Normalization of the column vectors of matrix B^k realizes as follows:

$\overline{B}^k = \left|\overline{b}_{i,j}^k\right| = \left|b_{i,j}^k / \sum_j b_{i,j}^k\right|$

Search process of normalized matrices of the key (critical) rlations $\overline{B}^k$ has a cyclic character. With growth of k matrix $\overline{B}^k$ stabilizes its value and verge towards a stable value $\overline{B}_{\lim}$.

All columns in the limit matrix will be identical and contain groups of elements of the sought vector of absolute priorities.

Below we list the main statements which are used for substantiation of the numerical method of absolute priorities of alternatives and criteria search.

Statement 1. Route with length more than n always contains loop/feedback

Statement 2. For k-degree of the normalized matrix M that is more than maximal simple cycle or path in graph length, it is valid: $M^k=M^{k+i}$.

Statement 3. Matrix power on the step when in result of the next multiplication operation resulting matrix is equal to obtained on the previous step one, will determine length of the graph maximal simple path or cycle.

Statement 4. When as a result of sequent squaring the matrix becomes identical to the obtained on a previous step one, it will include all simple paths and cycles of the graph.

Statement 5. $M^n=M^{n+i}$. Therefore solution can be found for not more then n matrix operations, and the alhorithm complexity estimates by the operations number of order $O(n^3)$.

4. Decision support system on basis of complex integration of ANP methods.

In this section we consider decision support system when object of analysis is examined using system-organized study process which supposes *two various interpritations of relations under investigation* in decision support model.

These two models correspond with opinions of two investigation participants and give possibility to comparte obtained on both models results which conform to different world views.

Model of such system doesn't pretend to the required adequacy ensuring in general case.

Proposed decision support system **KeyANP** is based on the system integration of two matrix methods of decision taking with dependences and feedbacks:

1. ANP (Analytic Network Process), when multicriterion choice is carried out reasoning from **sum/prod** scheme;
2. Key factors of Analytic Network Process method with **max/prod** ranking technique.

Case study of decision making.

Examined problem can be considered as a standard because it is traditionally described in Saaty's works.

It is necessery to choose a car taking into account three criteria: **C**ost, C, **R**epareCost, R and **D**uration, D. Three alternatives are available: cars of producers **A**, **E** and **J**.

Relations between objects (criteria and alternatives) are given by pair comparisons coefficient $a_{ij}\geq 1$, where i - column , j – row of the matrix. In Tables 1 – 3 pair relations between elements of preference matrices and corresponding to them preference graphs are given. Indices **C, R** and **D** belong to criteria, while **A**, **E** and **J** belong to alternatives. For example, $a_{AE}{}^{c}=5$ in Table 1 indictes that the price of car produced by **A** is 5 times higher than the one produced by **E**. In the tables vectors of absolute priorities **w** and their normalized values are given. They are calculated by multiplicative critical path method. There also eigenvectors of preference matrices **s** formulated by Saaty's method given for comparison.

Computed normalized vectors of relative priorities are represented by supermatrix in Table 7. It contains of two cellular matrices with normalized rows. Value $W_{CA}=0{,}6923$ written in the cell (C,A) indicates that cost criterion weight for object A equals to $W_{CA}=0{,}6923$, and $W_{AC}=0{,}666667$ say that for alternative A cost for it takes a first place.

Raising initial matrix to a power and observing the process of its values establishment we stop on the sixteens step for **ANP-sum/prod** method (Table 8). Limit matrix search process for **ANP-max/prod** method is represented in Table 9.

Computations results are illustrated in Figure 1. Diagram analysis let us see the difference in priorities values for two examined decision making models. Both models have a practical value, and their common consideration let meke conclusions which are conditioned by the system approach to decision making problem on basis of model approaches set.

The results enable us to make decision:

- We choose a car of producer A because it agrees with maximal absolute priority, that is almost twice as much as weighed characteristics of producers of the other cars. The second position takes a car of producer J.
- Cost is dominating choice criterion, on the second position is value of the weighed criterion D.

Table 1. Relations between costs

	A	E	J	w_j		s
A		5	3	9	0,6923	.637
E				1	0,0769	.105
J		3		3	0,2307	.258

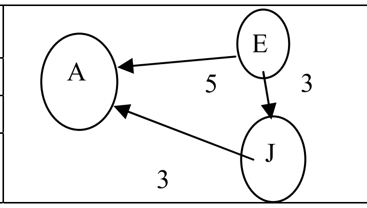


Table 2. Relations between repare costs

	A	E	J	w_j		S
A		5	2	6	0,6	.582
E				1	0,1	.109
J		3		3	0,3	.309

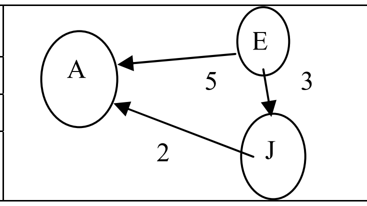


Table 3. Relations between durations

	A	E	J	w_j		s
A				1	0,0769	.105
E	5		3	9	0,6923	.637
J	3			3	0,2307	.258

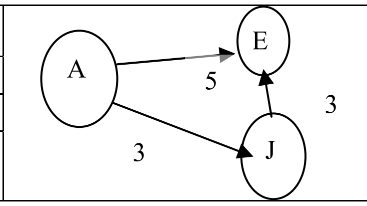


Table 4. What we value in cars A, comparing their criteria

	C	R	D	w_j		s
C		3	4	4	0,6667	.634
R			1	1	0,1667	.192
D		1		1	0,1667	.174

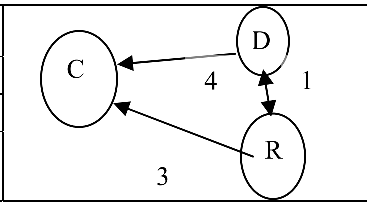


Table 5. What we value in cars E, comparing their criteria

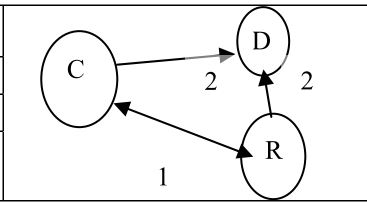
	C	R	D		w_j		s
C		1			1	0,25	.25
R	1				1	0,25	.25
D	2	2			2	0,5	.5

Table 6. What we value in cars J, comparing their criteria

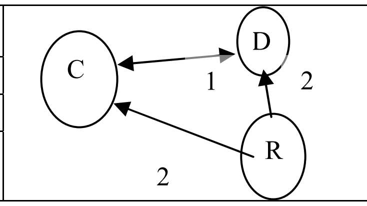
a_{ij}^{J}	C	R	D		w_j^{J}		s
C		2	1		2	0,4	.4
R					1	0,2	.2
D	1	2			2	0,4	.4

Table 7. Initial supermatrix

	CRITERIA			ALTERNATIVES		
	C	R	D	A	E	J
C	0	0	0	0,666667	0,25	0,4
R	0	0	0	0,166667	0,25	0,2
D	0	0	0	0,166667	0,5	0,4
A	0,6923	0,6	0,076923	0	0	0
E	0,0769	0,1	0,692308	0	0	0
J	0,2307	0,3	0,230769	0	0	0

Table 8. Search of limit matrix by ANP-sum/prod method

	0.573077	0.545	0.316667	0	0	0
	0.180769	0.185	0.232051	0	0	0
	0.246154	0.27	0.451282	0	0	0
A^2	0	0	0	0.574359	0.361538	0.427692
	0	0	0	0.183333	0.390385	0.327692
	0	0	0	0.242308	0.248077	0.244615

	0.504885	0.498652	0.450848	0	0	0
	0.194157	0.195398	0.204894	0	0	0
	0.300958	0.30595	0.344258	0	0	0
A^4	0	0	0	0.499803	0.454893	0.468742
	0	0	0	0.256272	0.299975	0.286495
	0	0	0	0.243925	0.245132	0.244763

	0.487412	0.487135	0.485005	0	0	0
	0.197629	0.197685	0.198108	0	0	0
	0.314959	0.315181	0.316887	0	0	0
	0	0	0	0.480717	0.478717	0.479334
A^8	0	0	0	0.274844	0.27679	0.27619
	0	0	0	0.244439	0.244492	0.244476
	0.486599	0.486599	0.486594	0	0	0
	0.197791	0.197791	0.197792	0	0	0
	0.31561	0.31561	0.315614	0	0	0
A^{16}	0	0	0	0.47983	0.479826	0.479827
	0	0	0	0.275708	0.275712	0.275711
	0	0	0	0.244462	0.244463	0.244462

Table 9. Search of limit matrix by ANP-max/prod method

	0	0	0	0.666667	0.25	0.4
	0	0	0	0.166667	0.25	0.2
A	0	0	0	0.166667	0.5	0.4
	0.692307692	0.6	0.076923	0	0	0
	0.076923077	0.1	0.692308	0	0	0
	0.230769231	0.3	0.230769	0	0	0
	1	1	1	1	1	1
	0.461538462	0.4	0.173077			
	0.115384615	0.1	0.173077			
	0.115384615	0.12	0.346154			
B^2				0.461538	0.173077	0.276923
				0.115385	0.346154	0.276923
				0.153846	0.115385	0.092308
	0.692307692	0.62	0.692308	0.730769	0.634615	0.646154
	0.213017751	0.184615	0.079882			
	0.053254438	0.046154	0.059911			
B^4	0.053254438	0.046154	0.119822			
				0.213018	0.079882	0.127811
				0.053254	0.119822	0.095858
				0.071006	0.039941	0.042604
	0.319526627	0.276923	0.259615	0.337278	0.239645	0.266272
	0.045376562	0.039326	0.017016			
B^8	0.011344141	0.009832	0.007179			
	0.011344141	0.009832	0.014357			
				0.045377	0.017016	0.027226
				0.011344	0.014357	0.011486
				0.015126	0.005672	0.009075
	0.068064844	0.05899	0.038552	0.071846	0.037046	0.047787

B^{16}	0.002059032	0.001784	0.000772			
	0.000514758	0.000446	0.000193			
	0.000514758	0.000446	0.000206			
				0.002059	0.000772	0.001235
				0.000515	0.000206	0.000309
				0.000686	0.000257	0.000412
	0.003088549	0.002677	0.001171	0.00326	0.001236	0.001956
B^{32}	4.23961E-06	3.67E-06	1.59E-06			
	1.0599E-06	9.19E-07	3.97E-07			
	1.0599E-06	9.19E-07	3.97E-07			
				4.24E-06	1.59E-06	2.54E-06
				1.06E-06	3.97E-07	6.36E-07
				1.41E-06	5.3E-07	8.48E-07
	6.35942E-06	5.51E-06	2.38E-06	6.71E-06	2.52E-06	4.03E-06

Normalized values

0.666667	0.645161	0.25	0	0	0
0.166667	0.16129	0.25	0	0	0
0.166667	0.193548	0.5	0	0	0
0	0	0	0.631579	0.272727	0.428571
0	0	0	0.157895	0.545455	0.428571
0	0	0	0.210526	0.181818	0.142857

0.666667	0.666667	0.307692	0	0	0
0.166667	0.166667	0.230769	0	0	0
0.166667	0.166667	0.461538	0	0	0
0	0	0	0.631579	0.333333	0.48
0	0	0	0.157895	0.5	0.36
0	0	0	0.210526	0.166667	0.16

0.666667	0.666667	0.441379	0	0	0
0.166667	0.166667	0.186207	0	0	0
0.166667	0.166667	0.372414	0	0	0
0	0	0	0.631579	0.45933	0.569733
0	0	0	0.157895	0.38756	0.240356
0	0	0	0.210526	0.15311	0.189911

0.666667	0.666667	0.65921	0	0	0
0.166667	0.166667	0.164802	0	0	0
0.166667	0.166667	0.175988	0	0	0
0	0	0	0.631579	0.624882	0.631579
0	0	0	0.157895	0.166823	0.157895
0	0	0	0.210526	0.208294	0.210526

0.666667	0.666667	0.666667	0	0	0
0.166667	0.166667	0.166667	0	0	0
0.166667	0.166667	0.166667	0	0	0
0	0	0	0.631579	0.631579	0.631579
0	0	0	0.157895	0.157895	0.157895
0	0	0	0.210526	0.210526	0.210526

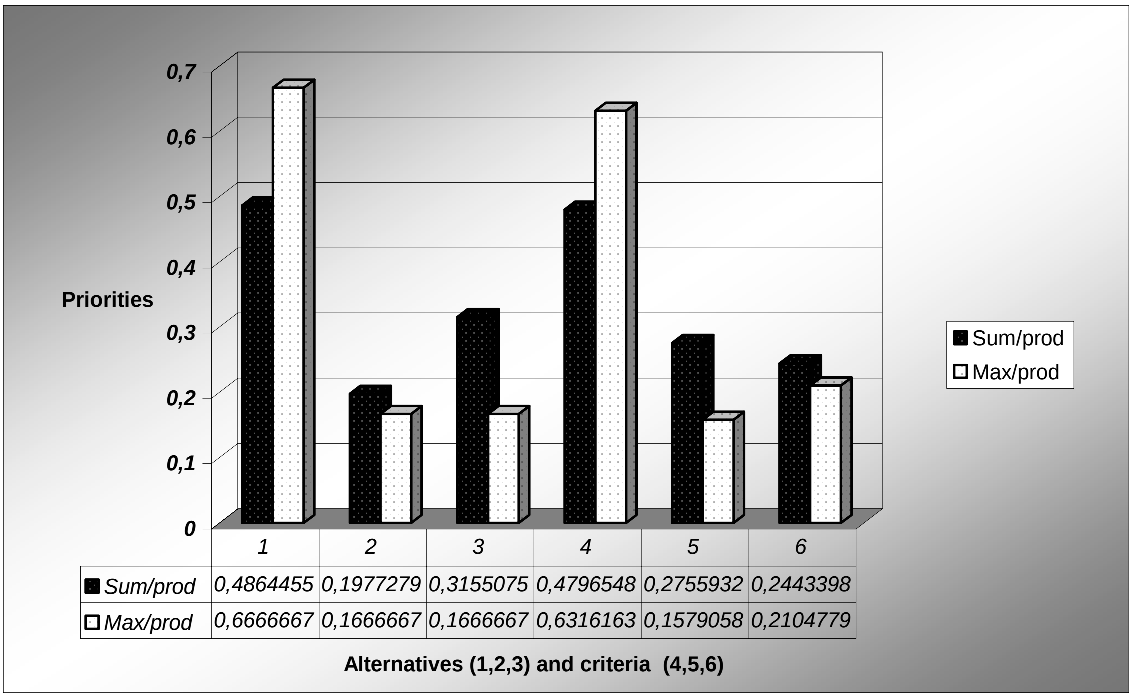


Figure 1. Comparing of priorities computation resultats by ANP-sum/prod и ANP-max/prod methods

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