A Mathematical Framework for a Unified Understanding of Mind and Matter:

The Analytic Network Process and its Generalization

Thomas L. Saaty

University of Pittsburgh

saaty@katz.pitt.edu

Abstract

The outcomes of the AHP/ANP when applied to the measurement of intangibles are normalized priorities. Priorities cannot be forced into a Cartesian system of coordinates. Science does not deal with intangibles and as a result uses Cartesian axes with arbitrary units to measure tangibles which in the end must always be interpreted. Where does the AHP/ANP with its generalization to stimulus response and the derivation rather than experimental discovery of natural law fit into the larger picture of scientific measurement?

Continuous comparisons are represented by Fredholm's equation of the second kind. A necessary condition for the solution of this equation is that a functional equation of proportionality, $w(as) = bw(s)$ should hold. Solutions of this functional equation in the real and in the complex domains provide us respectively with the inverse square natural law and about Dirac type distributions that represent firings of neurons in response to stimuli. Generalization of the equation to a proportionality functional equation in operators gives us solutions that determine the patterns of all responses to stimuli including the patterns or forms mathematics takes when applied in science. A pair of similar equations that involve dependence between stimulus and response have been recently solved.

Keywords: Structures in AHP/ANP, generalization of ANP to continuous judgments, functional equation of proportionality, Cartesian coordinate and hierarchies, influence, mind and matter, intangibles, relative measurement

Introduction

It is safe to say that the use of mathematics in science has a certain intrinsic weakness in its assumptions about how things began, how they are and how they will be. The universe consists of a set of influences. Because of dependence and feedback influences can be thought of as effects which themselves serve as influences that bring about other effects. In the end, we are only concerned with the relative intensities of the relevant effects, contextual to our specific problem, as they would directly affect the relative preference among some feasible courses of action.

Our theories and knowledge in science are based on numbers obtained from measurement with respect to properties like length, mass, temperature and time; criteria known as tangibles. The measurement of tangibles always involves the use of an arbitrarily chosen unit. The measurement uses multiples and fractions of that unit and must always be interpreted as to its relevance and significance to serve some purpose. But there is a myriad of intangibles we have no scales to measure. Some people have tried to create measurements for them in terms of some common unit like money, but they do not all relate to economic value except by a great and unreliable stretch of the imagination. How are our minds equipped to deal with intangibles and then relate them in a meaningful way to the tangibles that we know how to measure?

Few people in science would subscribe to the need to include intangibles as an important part of scientific explanation. Had not the AHP as a way of measuring intangibles been shown to be a powerful tool in decision making, speaking about intangibles and their absence in science would have seemed as hubristic gibberish. We must tread in this area with delicacy and care. There are many more things that we don't know how to measure than things we know how to measure. There are also many influences that have a very long duration in time while other influences are very transient and temporary.

The object of this talk is first to highlight the process of measuring intangible factors with some of its consequences in mathematics and science and second to make comparisons and distinctions about its uses in decision making as a tool of synthesis instead of the analytical, Cartesian based formulas and descriptive tools of science that ignore intangibles in their considerations, or for that matter they know how to measure every intangible they need in their theories

Measurement

Long before measurement scales were invented, people had no direct way to measure because they had no scales and had to compare things with each other or against a standard to determine their relative order. We still have that ability, and it is still critically necessary to be able to make comparisons much of the time, especially when we cannot measure things. One reason is that we do not have an instrument or scale to do it. Another reason is that even when we have a scale, the unit used is arbitrary and requires experience to interpret its reading. Expert judgment is needed to tell us about the importance of the numbers obtained and for what purpose. In that sense science is ultimately subjective.

History records that one day, René Descartes noticed a fly crawling around on the ceiling. He watched the fly for a long time. He wanted to know how to tell someone else where the fly was. Finally he realized that he could describe the position of the fly by its distance from the walls of the room. When he got out of bed, Descartes wrote down what he had discovered. Then he tried describing the positions of points, the same way he described the position of the fly. Descartes had invented the coordinate plane! In fact, the coordinate plane is sometimes called the Cartesian plane, in his honor.

One has to leave out so much reality to get to Descartes' beautiful attempt at representing the world. He was trying to assign measurements so that people could deal with things in a repeatable way. In a Cartesian world one first measures and then applies judgments. A number derived from a scale of measurement with an arbitrary unit like 50 yards does not suggest anything to a person who never heard of yards and has no idea how long a yard is. Even if he did understand the concept of a yard, he would have to answer the question about how large 50 yards is and what purpose it serves. Such questions require interpretation according to meaning which is more subjective and philosophical than objective and oriented towards an eternal truth.

As purposeful beings we have a sense of priority and order. Priorities can be assigned an arbitrary unit but they are not objective and differ from one person to another person. They fall on an n-dimensional tetrahedron. People have tried and still try to derive priorities by using metric properties involving measurements. But only a small set of things is measureable. Most factors or criteria are intangible and have no scales of measurement. They must be compared directly to derive their priorities.

There are two ways to measure anything, the first is by using a scale of measurement that may be constructed physically like the pound scale for measuring weight or the yard stick for measuring length, or mentally like measuring intelligence or IQ, and the second by deriving a scale of measurement. The first is a familiar process, but the second is what needs to be discussed in detail, which we do here. Because of Cartesian coordinates, physics and astronomy are riddled with dimensions and theories about dimensions. Every time a new factor is thought to have influence a new dimension is added. There has to be something fundamentally missing in physics despite the fact that brilliant minds have developed the subject not unlike those who championed religions in the name of eternal truth.

We take it for granted in mathematics that when we have a problem to solve, we can use Cartesian axes to represent independent variables that have numbers attached to them and proceed to define functions in terms of these variables and happily manipulate these functions according to established rules. This approach is motivated by the original symbiotic relationship between mathematics and physics where nearly everything has already been reduced to the measurement of length, mass and time. We simply ignore those properties that are intangible and need judgment to measure and speak of infinite dimensional spaces both countably and uncountably infinite. It is seldom that we inquire whether there may not be other independent variables that we need for which there are no known measurements and can never be represented on a system of axes. The world is full of such "intangible" variables - more than the measurable ones known as tangibles.

The Compelling Need to Make Comparisons

People use their judgment to order things. The way they do it is to compare two things at a time to determine which was the better or more preferred with respect to a common property. By repeating the comparisons they obtain a total ordering of the objects without assigning them numerical values specially when there are no numbers to conveniently assign. After being ordered on a property, they are ranked first, second, and so on. But when many properties are involved it is no longer easy to combine the orders obtained by assigning numbers from different scales to obtain a total order. That is what modern decision making tries to do.

How can we measure things in a way that captures their influence on one another? We need numbers that do not need a unit and an origin in their definition. What are such numbers? We need absolute scales of numbers which do not need to be defined in terms of a unit or an origin. An absolute number means how many times more. So for a pair, let the lesser element be the unit and use an absolute number to indicate how many multiples of that unit is the larger element.

The paired comparisons process using actual measurements for the elements being compared takes the form:

$$
\begin{array}{c c}
 & \begin{array}{cccc} A_1 & A_2 & \cdots & A_n \\ w_1 & w_2 & \cdots & w_n \end{array} \\
\begin{array}{c} A_1 \\ A_2 \\ \vdots \\ A_n \end{array} & \begin{bmatrix} w_1/w_1 & w_1/w_2 & \cdots & w_1/w_n \\ w_2/w_1 & w_2/w_2 & \cdots & w_2/w_n \\ \vdots & \vdots & \cdots & \vdots \\ w_n/w_1 & w_n/w_2 & \cdots & w_n/w_n \end{bmatrix}
\end{array}
$$

We note that we can recover the vector $w = (w_1, ..., w_n)$ by solving the system of equations defined by:

$$
Aw = \begin{bmatrix} 1 & a_{12} & ... & a_{1n} \\ 1/a_{12} & 1 & ... & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ 1/a_{1n} & 1/a_{2n} & ... & 1 \end{bmatrix} \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} = cw
$$

For a given positive matrix *A*, the only positive vector *w* and only positive constant *c* that satisfy *Aw = cw*, is a vector *w* that is a positive multiple of the principal eigenvector of *A*, and the only such *c* is the principal eigenvalue of *A*.

As a result of using coordinate systems, the idea of many dimensions is central in mathematical thinking even though we live in a dimensionally limited physical universe of space with time added as a fourth variable for convenience of interpretation. It appears that the measurement of intangibles subsumes the measurement of tangibles which may themselves be used simply to represent otherwise non-measurable intangibles. To reduce the measurement of tangibles on a ratio scale to a set of priorities, one can simply normalize these measurements rendering them dimensionless and putting them on par with the priorities of intangibles. It seems inevitable that understanding the world more fully by considering all the factors, one needs to use the AHP/ANP.

People with knowledge and awareness often find it difficult to exclude intangibles on the ground of technical difficulty. The quantum physicist David Bohm wrote: "The question is whether matter is crude and mechanical or whether it gets more and more subtle and becomes indistinguishable from what people have called mind." Arthur Eddington writes, "To put the conclusion crudely - the stuff of the world is mind-stuff." Einstein said,"We thus arrive at a conception of the relation of science to religion very different from the usual one...I maintain that the cosmic religious feeling is the strongest and noblest motive for scientific research." Charles Reich wrote, "The great and urgent need of these times is transcendence. The last two hundred years have fundamentally and irrevocably altered the terms of man's existence. The price of survival is an appropriate consciousness and social order to go along with the revolution of science and technology that has already occurred. The chaos we are now experiencing is the predictable and inevitable consequence of our failure to rise to this necessity...what is called for is a higher logic and a higher reason. The creation of a new consciousness is the most urgent of (our) real needs." According to Swami Muktananda, "To have the awareness that everything is made of one conscious energy is not only the highest science but the highest religion. No matter what we accomplish in the world, if we do not achieve this awareness of equality, none of it will be of any use." The AHP addresses this challenge.

AHP/ANP and How Our Brains Work?

All things and all experience is known to us through neural firings and therefore the entire universe is mapped into out brain that way and is subject to its refinement and limitations. We have no other way to understand anything beyond our instrument of understanding, our nervous system and our brain. The thought process is a higher level response activity to all the responses which themselves serve as second order stimuli for the thinking brain to analyze and respond proportionately. Every stimulus is the product of some other stimulus and must satisfy conditions imposed by that stimulus. There are two possibilities. Either all stimuli are interdependent and there is no first stimulus, or there is a first stimulus whose origins cannot be explained. We who think about and practice the ANP believe that the former situation of interdependence is the more likely situation. It is also in agreement with what cosmologists tell us about the origins of the universe and about the Big Bang, although perhaps not with the way it has been interpreted due to limitations in our reasoning with linear logic about a process that is by its very nature cyclic.

Proportionality with respect to a single stimulus requires that our response to a proportionately amplified or attenuated stimulus we receive from a source should be proportional to what our response would be to the original value of that stimulus. If $w(s)$ is our response to a stimulus of magnitude s, then the foregoing gives rise to the functional equation $w(as) = b\, w(s)$. This equation can also be obtained as the necessary condition for solving the Fredholm equation of the second kind:

$$\int_a^b K(s,t)\,w(t)\,dt = \lambda_{\max}\,w(s)$$

obtained as the continuous generalization of the discrete formulation $Aw = \lambda_{\max} w$ for obtaining priorities where instead of the positive reciprocal matrix A in the principal eigenvalue problem, we have a positive kernel, $K(s,t) > 0$, with $K(s,t)\ K(t,s) = 1$ that is also consistent i.e. $K(s,t)\ K(t,u) = K(s,u)$, for all s, t, and u. The solution of this functional equation in the real domain is given by

$$w(s) = Ce^{\log b \frac{\log s}{\log a}} P\left(\frac{\log s}{\log a}\right)$$

where P is a periodic function of period 1 and P(0) = 1.

The logarithmic law of response to stimuli can be obtained as a first order approximation to this solution through series expansions of the exponential and of the cosine functions as:

$$v(u) = C_1 e^{-\beta u} P(u) \approx C_2 \log s + C_3$$

$\log ab \equiv -\beta, \beta > 0$. The expression on the right is known as the Weber-Fechner law of logarithmic response $M = a \log s + b, a \neq 0$ to a stimulus of magnitude s. This law was empirically established and tested in 1860 by Gustav Theodor Fechner who used a law formulated by Ernest Heinrich Weber regarding discrimination between two nearby values of a stimulus. We have now shown that it can be derived that Fechner's version can be derived by starting with a functional equation for stimulus response.

The integer-valued scale of response used in making paired comparison judgments can be derived from the logarithmic response function as follows. For a given value of the stimulus, the magnitude of response remains the same until the value of the stimulus is increased sufficiently large in proportion to the value of the stimulus, thus preserving the proportionality of relative increase in stimulus for it to be detectable for a new response. This suggests the idea of just noticeable differences (jnd), well known in psychology. Thus, starting with a stimulus s_0 successive magnitudes of the new stimuli take the form:

$$s_1 = s_0 + \Delta s_0 = s_0 + \frac{\Delta s_0}{s_0} s_0 = s_0(1+r)$$

$$s_2 = s_1 + \Delta s_1 = s_1(1+r) = s_0(1+r)^2 \equiv s_0\alpha^2$$

$$\vdots$$

$$s_n = s_{n-1}\alpha = s_0\alpha^n \ (n = 0,1,2,\ldots)$$

We consider the responses to these stimuli to be measured on a ratio scale (*b*=0). A typical response has the form $M_i = a\log\alpha^i$, *i* =1,…,*n*, or one after another they have the form:

$$M_1 = a\log\alpha, M_2 = 2a\log\alpha, ..., M_n = na\log\alpha$$

We take the ratios $M_i / M_{1,}$ i = 1,…,n of these responses in which the first is the smallest and serves as the unit of comparison, thus obtaining the *integer* values 1, 2, …, n of the fundamental scale of the AHP. It appears that numbers are intrinsic to our ability to make comparisons, and that they were not an invention by our primitive ancestors. We must be grateful to them for the discovery of the symbolism. In a less mathematical vein, we note that we are able to distinguish ordinally between high, medium and low at one level and for each of them in a second level below that also distinguish between high, medium and low giving us nine different categories. We assign the value one to (low, low) which is the smallest and the value nine to (high, high) which is the highest, thus covering the spectrum of possibilities between two levels, and giving the value nine for the top of the paired comparisons scale as compared with the lowest value on the scale. In fact we show later that because of increase in inconsistency when we compare more than about 7 elements, we don't need to keep in mind more than 7 ± 2 elements. This was first conjectured by the psychologist George Miller in the 1950's and explained in the AHP in the 1970's. Finally, we note that the scale just derived is attached to the importance we assign to judgments. If we have an exact measurement such as 2.375 and want to use it as it is for our judgment without attaching significance to it, we can use its entire value as it is.

We can think of thoughts themselves as stimuli. First order stimuli related to nature lead to natural laws and second order stimuli lead to the laws of thought, because thought are themselves stimuli both for acting on nature and for stimulating other thought in response. We go from a stimulus that is a variable, to response to that stimulus in the form of a judgment which can be represented as a function and now move to higher order response (a thought) to a response function which can be represented as an operator on functions. An "operator" is a transformation that applies to a set of functions rather than to a set of points, a "function of functions". We have developed the theme of proportionality among responses to stimuli and moved deeper into proportionality among the responses themselves. As in group decisions, we need to combine responses registered in memory into a single overall response by synthesizing them according to their importance. It is not difficult to see that we need that because we need to be rational, connected with non-reducible logic, to explain our diverse responses. Here again we have two ways to justify our approach. The first is to justify the need to solve a general proportionality functional equation in operators $W[af(x)] = bW[f(x)]$ and the second is to derive this condition as a necessary condition for solving an operator equation that is a generalization of Fredholm's equation.

What the New Theory of Behavior Should be Like

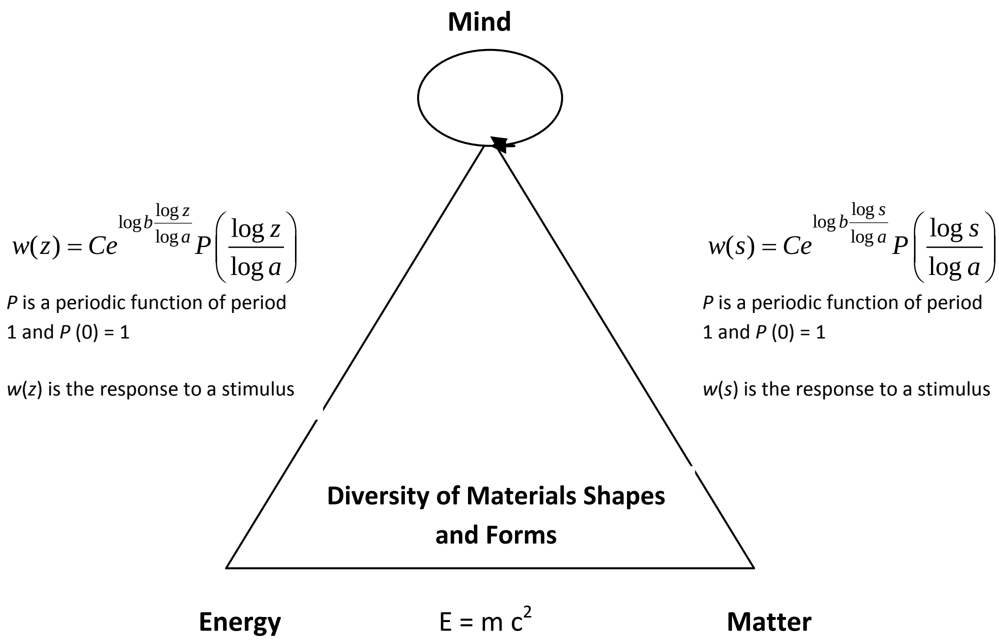
Modern Mathematics is structured around an objective world assumed to be closed that is independent of the observer. Geometries (e.g. metric, Euclidean, affine, and projective) are defined in terms of invariance under a group of transformations of which Lie groups are the deepest studied and most used to study the physical world. The objects under a transformation are only changed according to that transformation and not by adapting themselves after many transformations to a new kind of object. Closure is a significant property under which all things are studied in mathematics. New and strange things outside the system are assumed not to occur. Mathematics as it is today is a very limiting instrument for an abstract understanding of psychological behavior. In particular, to relate things in quantitative terms involving measurement of different intensities of behavioral response is not possible in the ordinary way in which objects in physics are measured on concrete scales according to extension and weight and their dynamic change with respect to time. Unlike physics and geometry with their shapes and weights that can be felt with the senses, in psychology things are studied according to their change in intensity and connection according to the influence of that intensity on other things. Our senses are an unreliable form of detecting intensity and influence. Psychology and politics require a higher form of abstraction. In addition, the interpretation of behavior has a relative form of evaluation according to the norms of those who study it because unlike the senses that are genetically inherited by all people, the mind has a considerable degree of flexibility to interpret things and can vary considerably in its interpretations. They are done relative to a purpose or goal that the observer has in mind. Thus by its very nature measurement with respect to a goal must be relative and not absolute with a unit defined to enable one to perform the measurement. By its flexible and open nature relative measurement cannot be interpreted within the framework of our existing geometries that are based on our use of the senses to interpret the physical world. The only thing it has to share with modern science is the use of numbers, not to measure intensities on an absolute scale, but to determine relations among intensities and their influence and also the use of the abstract idea of transformations which may themselves not be subject to closure, and may change and evolve over time. Interestingly, the two outlooks share the idea of equilibrium and stability with remarkable clarity. If things are subjected to transformation do they converge to a stable outcome, oscillate or diverge to an intractable form? Finally, with behavior one needs to order properties according to the priority of their influence. Although metric properties are often needed, order properties take precedence over them. What has higher priority is of greater interest than what is close to what (order topology rather than metric topology). The elements of a psychological or behavioral system are *open* to integration with other elements to form a group of elements with its own group behavior and not as a collection of objects. Each element has its own survival properties plus its interaction within a larger system of people. These two drives interact and influence one another. Systems of people combine into larger ones and so on so the groups develop attitudes in the individuals that may not have been originally there. Conflict resolution and harmony by ordering priorities with respect to conflicting goals and the priorities of these goals with respect to still higher goals within an individual, between individuals and among groups to act on them and allocate resources in a consistent way becomes a major concern in the social field.

It is too early in the history of the physical universe to combine physical and behavioral forces in a way that the latter can exhibit a significant degree of universal effect. But such mathematical considerations are possible and are in conformity with human mental expectations that there is more to this universe than physical force. It is assumed that light travels at uniform speeds through all distances large or small and so do other natural phenomena behave homogeneously so we can cope with understanding their complexity. Proportionality is the invariant of the social sciences, and we have proportionate response behavior. People react or respond contextually in proportion to expectations, theirs and others' developed from previous experience and from taught social norms. This is a fundamental principle.

Through its influences is how we understand it

The current evolutionary interpretation of what happens in the real world is that matter and energy with uncertain origins and unknown structures bring about mind. All the influence flows in one direction, from nature to mind. The idea is that things happen at random through natural selection and that the mind is one of the consequences of evolution and its trial and error varied combinatorial approach, a result of interactions of the chemistry and electricity of the brain. The mind is only consequential in that it helps its owner to survive by controlling the body and assisting in the search for food. Its abstractions are its ways to look for order and to anticipate the future in order to survive its hazards, by organizing and classifying possible structures or scenarios to design strategies that enhance survival. The mind can have no intrinsic influence on nature in the large to change its laws and events. Nevertheless mind is in nature and creates meanings and abstractions that are organized and fall in the domain of the laws of nature which we can at best say today that it is the domain of energy that belongs to mathematical scientific thinking. It is the third contender in the energy, matter and mind complex system. There are numerous manifestations of mind and its scientific understanding elaborated by other forms of existence, like orbital atoms, simple forms of life and complex forms of life exhibited plant and animal forms. The older and opposite view was that an intelligent mind brought about the natural world, its matter, energy and all its cosmic influences and interactions.

If we were to consider energy, matter and mind to be interdependent, and that the origins of the universe whether they began with a big bang or otherwise did not happen at once but took time no matter how small to arrange their happening, then we might argue that mind is a third form that is a party to what happens in the system. There may eventually be other kinds of essence in addition to energy, matter and mind that are inherently there but take time to occur through emergence and synergy, and that eventually they all interact and expand, increasing their kind or contract to nothingness. We may assume that despite the time lag these emergent forces are interdependent just as we once thought in relativistic terms that matter and energy were interchangeable. It appears that in its evolution, the universe likes to have its story recorded and told through a memory akin to that of the mind. It also has its way of communicating to the mind through the senses and other influences its substance and behavior that in turn enables the mind to grasp and improve its sensitivity to tell a richer and richer story. In a sense the universe is an actor on the stage of the mind. Consequently, how the mind grasps and tells the story is known in through the mind itself that must continuously change to take in the whole story. It is an unspoken intuitive dialogue that is as real as anything we can use to account for our zest for knowledge and dedication to learning at high cost and inconvenience particularly in the realm of physics, biology, astronomy and psychology and neurology.



Cartesian thinking teaches us that there is unity in knowledge that can be reduced to mathematics in a deductive way and that we can axiomatize and deductively arrive at a coherent and consistent explanation of all reality in a logical way. It is easy to note that when there is dependence and feedback, deductive logic is dysfunctional and inadequate for explaining complexity because of its linearity in going from causes to effects without a systematic procedure to allow for feedback. The AHP teaches us that there are different perspectives of the world that can be combined by using the judgments of different people and that the outcome of such a synthesis needs to take into account how strongly these people represent the intensity of influences that produce a certain outcome. That intensity is recorded quantitatively by our nervous system. But if by adding more people we can change the outcome, then unity of perspective associated with the Cartesian view, beautiful as it is, cannot be achieved. We deceive ourselves in thinking that there is an objective view that with adequate research and exploration we can develop and use to explain everything. It is an old paradigm that is unworkable for our time with its increasing complexity. At best we can hope to explain how our thinking brains and nervous system work and use that understanding to determine how it processes information and represent that information with the tools we use to explain its behavior, namely electrical signals and their syntheses. We believe that all we can ever be aware of, feel, describe, understand and communicate are all represented by neural firings and synthesis, whether we do it individually or synthesize it collectively as in the AHP/ANP. Because electric signals are quantitative and are synthesized quantitatively to make more meaning, for us the world is ultimately a combination of electric signals understood with mathematics, and there is no other way that it can be known to us. Our conclusion is that the world is in fact ultimately Cartesian.

Appendix

Table Summary of Discrete and Continuous Comparisons and Their Mathematical Implications

Discrete	**Continuous**
$A=(a_{ij})$, A consistent that is $a_{ij}a_{jk}=a_{ik}$, then $a_{ij}=w_i/w_j$ and $\sum_{j=1}^{n} a_{ij}w_j = nw_i,\ i=1,\dots,n,\ \sum_{i=1}^{n} w_i = 1$	$K(s,t)$ consistent that is $K(s,t)K(t,u)=K(s,u)$ then $K(s,t)=K(s)/K(t)$ and $\lambda\int_a^b K(s,t)w(t)dt = w(s),\ \int_a^b w(s)ds = 1$
$A=(a_{ij})$, A reciprocal that is $a_{ji}=1/a_{ij}$ but not consistent, then $\sum_{i=1}^{n} a_{ij}w_j = \lambda_{\max}w_i,\ \sum_{i=1}^{n} w_i = 1$	$K(s,t)$ reciprocal that is $K(s,t)K(t,s)=1$, but not consistent, then $\lambda\int_a^b K(s,t)w(t)dt = w(s),\ \int_a^b w(s)ds = 1$
Principal eigenvector solution of $Aw=\lambda_{\max}w$	Eigenfunction solution also solution of functional equation $w(as)=bw(s)$; $w(s) = b^{\log s/\log a}p(\frac{\log s}{\log a})$ or more simply $w(u)=e^{\alpha u}p(u)$, $\alpha=\log b$, $u=\log s / \log a$. In the complex domain a single valued solution of the functional equation: $w(az)=bw(z)$, is $w(z)=cb^{\log\lvert z\rvert/\log\lvert a\rvert}g(z)$ where $g(z)$ is an arbitrary solution of $g(az)= g(z)$.
	Fourier Transform of real solution: $C\frac{\beta}{\sqrt{2\pi}}\left[\frac{1}{\beta^2+\left(\frac{1}{2\pi}+\xi\right)^2}+\frac{1}{\beta^2+\left(\frac{1}{2\pi}-\xi\right)^2}\right]$ When the constants in the denominator are small relative to ξ we have C_1/ξ^2 which we believe is why optics, gravitation (Newton) and electric (Coulomb) forces act according to inverse square laws. Fourier Transform of complex solution: $F(x) = (1/2\pi)\log a\sum_{n=-N}^{N} a_n^{'}\frac{(2\pi n+\theta(b)-x)}{\log a\lvert b\rvert+(2\pi n+\theta(b)-x)}i\delta(2\pi n+\theta(b)-x)$

	With a Dirac delta function signifying impulsive firings to stimuli as the neurons of the brain do.
Hierarchic Composition gives rise to multilinear forms $w_i^h = \sum_{i_2,\ldots,i_{h-1}=1}^{N_{h-1},\ldots,N_1} w_{i_1 i_2}^{h-1} \ldots w_{i_{h-2} i_{h-1}}^{2} w_{i_{h-1}}^{1} \quad i_1 \equiv i$	Hierarchic composition in the case of a finite number of criteria is a particular case of the multiple stimuli solution. It is a product integral in the case of a continuum number of criteria, as in the case of a continuum number of stimuli. For a **continuum number of stimuli** let $K(\mathbf{X},\mathbf{Y})$ be a compact consistent kernel i.e. $K(x,y)K(y,z)=K(x,z)$, for all $x\in\mathbf{X}$ and $y\in\mathbf{Y}$ and $z\in\mathbf{Z}$, where ***X*, *Y* and *Z*** are compact subsets of the reals. We have the equation $w(\mathbf{X}) = \lambda\int_{\Omega} K(\mathbf{X};\mathbf{Y})w(\mathbf{Y})$. Solution of this equation involves not simply distributions, but also Lebesgue integrals of distributions.
Network composition also gives rise to convergent series of multilinear forms	Continuum composition- same as with hierarchic composition
	Operator Equation from stimulus to response **$W(\alpha X)=\beta W(\mathrm{X})$, Completely solved (see book page 505)**
	Interdependence of stimulus and response like mind and matter, God and creation yields a system of two functional equations in operators: $W(\alpha\mathbf{X}) = \beta W(\mathbf{X})$ and $\mathbf{X}(\beta W) = \alpha\mathbf{X}(W)$