AN INDUCTION BASED ON A HYBRID OF DRSA AND DEMATEL FOR ANALYZING COMPETITIVENESS 2012

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ABSTRACT

Faced with the worldwide challenges, the interdependence among criteria deserves paying attention for national competiteness. When considering the influence in others and the sensitivity to others, this issue becomes complicated and hard. In this research, a hybrid of DRSA and DEMATEL is proposed to generate an interdependence map by the induction information, which helps in understanding the inside of competitiveness. For an empirical illustration, the proposed method is applied to analyze on World Competitiveness Yearbook 2012. The result shows that institutional framework, health and environment, and basic infrastructure are significant to other criteria.

Keywords: competitiveness, dominance-based rough set approach (DRSA), Decision-Making Trial and Evaluation Laboratory (DEMATEL)

1. Introduction

Competitiveness has been playing an aggregated power of a nation to enhance people’s lives and cope with worldly challenges (U.S. President’s Commission on Industrial Competitiveness, 1985; WEF, 2003; IMD, 2012). National leaders need pay careful attention not only to competition but also to interdependence among criteria which can help and enhance policy-making. There are two attractive points in the criteria interdependence. One is the influence of a criterion in others. The other is the sensitivity of a criterion to others. This research proposes a network relationship map (NRM) of criteria interdependence based on a hybrid of DRSA and DEMATEL (HDD). The hybrid can present the knowledge of influence and sensitivity among criteria by induction rules. Empirically, the proposed method is applied on the World Competitiveness Yearbook (WCY) in 2012. The result reveals significant criteria with the influence and sensitivity in NRM.

The remainder of this paper is organized by reviewing national competitiveness, DRSA, and Decision-Making Trial and Evaluation Laboratory (DEMATEL) in Section 2, proposing the hybrid method in Section 3, describing the application results in Section 4, and discussing and concluding the paper in Section 5.

2. Literature review

DRSA is a powerful technique of relational structure and has been successfully applied in many fields (Greco et al. 2000, 2001, 2002). In classification application, it can be used to induce objects assigned to
Cl^t_e (the upper ward union classes which include objects ranked at least \( t^{th} \) ) or to Cl^t_w (the downward union of classes which includes objects ranked less than \( t^{th} \)), where Cl is a cluster set containing ordered classes Cl_t, \( t \in T \) and \( T = \{1, 2, ..., n\} \). The formulations for the above statement can be expressed as 
\[ Cl = \{Cl_1, ..., Cl_t, ..., Cl_n\}, \quad Cl_t = \{y \in U : y \text{ is ranked in the top position}\}, \quad Cl_{t+1} = \{y \in U : y \text{ is ranked in the second position}\}, \ldots, \quad Cl_n = \{y \in U : y \text{ is ranked in the bottom position}\} \] where \( U \) is a set with decision makers’ preference orders. For all \( s, t \in T \) and \( s \geq t \) (rank of \( s \geq \) rank of \( t \)), every object in \( Cl_s \) is preferred to be at least as good as any of object in \( Cl_t \). They are constructed as:

The dominating union: \( Cl^t_s = \bigcup_{s \geq t} Cl_s \) for \( s \geq t \) ; the dominated union: \( Cl^t_s = \bigcup_{s < t} Cl_s \) for \( s < t \).

Another representation of the dominating set relies on a set of criteria, \( P \). It follows the dominance principle of requiring each chosen object at least as good as a boundary object \( x \) in all considered criteria. The granules of a dominating set based on \( P \) can be viewed as the granular cones in the criteria value space. Vice versa the dominated sets follow the dominance principle and have granules in the opposite direction. These cones are categorized into \( P \)-dominating and \( P \)-dominated sets [26], respectively. It is said that object \( y \) \( P \)-dominates object \( x \) with respect to a criteria set \( P \) (denotation \( yD_px \)). Given \( x, y \in U \) and \( P \), let dominance sets as:

\( P \)-dominating set: \( D^+_p(x) = \{y \in U, yD_px\} \) and \( P \)-dominated set: \( D^-_p(x) = \{y \in U, xD_py\} \) where \( x, y \in Cl \), \( x \) plays a role for the boundary of \( D^+_p(x) \) or \( D^-_p(x) \), \( y \succeq x \) for \( D^+_p(x) \), \( x \succeq y \) for \( D^-_p(x) \), and all \( q \in P \).

Two approximations are defined for illustrating the dominance consistency. The association between \( Cl^t_s \) and \( P \)-dominating set should keep dominance consistency requiring \( y \in Cl^t_s \) and \( y \in P \)-dominating.

\[
P(Cl^t_s) = \{x \in Cl^t_s, D^+_p(x) \subseteq Cl^t_s\}, \quad \overline{P}(Cl^t_s) = \bigcup_{x \in Cl^t_s} D^+_p(x), \quad Bnp(Cl^t_s) = \overline{P}(Cl^t_s) - P(Cl^t_s)
\]

\[
P(Cl^-_s) = \{x \in Cl^-_s, D^-_p(x) \subseteq Cl^-_s\}, \quad \overline{P}(Cl^-_s) = \bigcup_{x \in Cl^-_s} D^-_p(x), \quad Bnp(Cl^-_s) = \overline{P}(Cl^-_s) - P(Cl^-_s)
\]

where \( t = 1, \ldots, n \), \( Bnp(Cl^t_s) \) and \( Bnp(Cl^-_s) \) are \( P \)-doubtful regions. \( P(Cl^t_s) \) is defined by requiring that the largest union of \( P \)-dominating sets should be included in \( Cl^t_s \). \( \overline{P}(Cl^t_s) \) is defined by requiring that the smallest union of \( P \)-dominating sets should contain all elements of \( Cl^t_s \). These two approximations present the proper and possible assignments of objects into \( Cl^t_s \) respectively. The objects belonging to the possible but not proper assignment are categorized as doubtful. The quality of DRSA can be explained with the coverage rate defined by Pawlak (1997, 2002) and Greco et al. (2000, 2001). There are two typical coverage rates (CR) for the upward union \( Cl^t_s \) and the downward union \( Cl^-_s \), which are formulated as:

\[
CR(Cl^t_s) = \frac{|P(Cl^t_s)|}{|Cl^t_s|} \quad \text{and} \quad CR(Cl^-_s) = \frac{|P(Cl^-_s)|}{|Cl^-_s|}
\]

The symbol \( CR \) is used to express “the probability of objects in the \( P \)-lower approximation relatively belonging to the corresponding union of decision classes.” The possible assignment can be explained by the accuracy rate. Two typical accuracy rates (\( \alpha \)) are listed as:

\[
\alpha(Cl^t_s) = \frac{|P(Cl^t_s)|}{|P(Cl^t_s)|} \quad \text{and} \quad \alpha(Cl^-_s) = \frac{|P(Cl^-_s)|}{|P(Cl^-_s)|}
\]

The symbol \( \alpha \) is used to present “a ratio of the cardinalities of \( P \)-lower approximation to those of \( P \)-upper approximation, i.e., the degree of the properly classified approximation relative to the possibly classified approximation.” The ratios can be operated into new ratio in mathematics, proposed by Saaty (2001).
The DEMATEL technique provides a comprehensive method for building and analyzing a structural model involving effective relationships among complex perspectives (Wu and Lee, 2007) and constructing the correlations between criteria to build a network relationship map, NRM (Tzeng et al., 2006; Liou et al., 2007). In addition, it has helped to develop the competencies of global managers (Wu and Lee, 2007), enabled socially responsible investment (Tsai et al., 2009) and assisted with cost evaluation in the hotel industry (Tsai et al., 2010). The followings present the hybrid of DRSA and DEMATEL to analyze national competitiveness in 2012.

3. Induction based on HDD (Hybrid of DRSA and DEMATEL)

3.1 Dataset
This research adopts the WCY dataset 2012 containing 59 nations, 4 consolidate factors, and 20 criteria.

3.2 The hybrid of DRSA and DEMATEL (HDD)
The hybrid of DRSA and DEMATEL starts from an information system of DRSA to a matrix of the criteria interdependence as the followings. From now on the objects in DRSA are termed as nations.

Proposition 1: Information system of DRSA
\[\text{DRSA} = (U, Q, f, V) \text{ where } U = \{ y_k | k = 1, \ldots, n \}, \quad Q = \{ q_1, q_2, \ldots, q_m \}, \quad \text{and } f : U \times Q \rightarrow V, \]
\[V_Q = (V_{q_1}, V_{q_2}, \ldots, V_{q_m}) \]. This proposition transforms sets into an information system.

Proposition 2: Preference orders
\[r_{ij} \geq r_{ij} \iff f(x, q_j) \geq f(z, q_j), \forall x, z \in U \text{ where } f \text{ is a function that maps a criterion to a preference value for a nation. For instance, } r_{ij} \text{ and } r_{ij} \text{ are preference values of nation } x \text{ and } z \text{ with respect to } q_j.\]

Proposition 3: A conditional dependent rules \(q_{ij, t'} \rightarrow q_{ij, t}^z\)
\(q_{ij, t'} \rightarrow q_{ij, t}^z\) represents how a criterion \(q_j\) conditionally depends on the top \(t\) positions of \(q_i\) where \(q_{ij, t'}^z\) is a set of nations within the top \(t'\) positions with respect to \(q_j\), \(q_{ij, t}^z\) is a set of nations within the top \(t\) positions with respect to \(q_j\), and \(t'\) and \(t\) are rank places. In this research \(t\) is set as \(10^6\) but \(t' \in \{1, 2, \ldots, n\}\). The approximations related to a rule can be conceptualized as Fig. 1. \(P(q_{ij, t}^z)\) is the lower approximation containing the boundary nation \(x\) and nations at least as good as \(x\) in all considered criteria in \(P\). \(\bar{P}(q_{ij, t}^z)\) is the upper approximation containing the boundary nation \(\bar{x}\) and nation at least as good as \(\bar{x}\) in all considered criteria. \(q_{ij, t}^z = \bigcup_{q_{ij, t}} q_{ij, t}\) contains nations ranked in at least \(t'\) with respect to criterion \(q_j\). So is \((q_{ij, t}^z = \bigcup_{q_{ij, t}} q_{ij, t})\).

Proposition 4: Approximations of \(q_{ij, t'} \rightarrow q_{ij, t}^z\).
\[P_{\mu, j} = \{ x \in q_{ij, t'}^z, D^v_{\mu}(x) \subseteq q_{ij, t}^z, P = \{ q_j \} \}, \quad \bar{P}_{\mu, j} = \bigcup_{x \in \bar{U}} D^v_{\mu}(\bar{x}), \]
\[Bnp_{\mu, j} = \bar{P}_{\mu, j} - P_{\mu, j}, \text{ and } P = \{ q_j \}. \] The boundary nations are presented as the slash lines.

Proposition 5: The accurate coverage rate (ACR) of \(q_{ij, t'} \rightarrow q_{ij, t}^z\) is formulated as \(g_{\mu} = g(q_{ij, t'} \rightarrow q_{ij, t}^z)\) which is a unique value to present the degree that \(q_j\) conditionally depends on the top \(t\) positions of \(q_i\), \(0 \leq g_{\mu} \leq 1\). It is formulated as Model I.

Model I: Max \( g_{\mu} = CR_{\mu} \times \alpha_{\mu} \)
s.t. \( P_{ji,t} = D^+_p(x), \quad \overline{P}_{ji,t} = D^+_q(x) \); \( CR_{ji} = \frac{|P_{ji,t}|}{|q_{ji,t}|}, \quad \alpha_{ji} = \frac{|P_{ji,t}|}{|\overline{P}(q_{ji,t})|} \)

where \( CR_{ji} \) represents a coverage rate of \( q_{ji,t} \rightarrow q_{ji,t} \). \( \alpha_{ji} \) represents an accuracy rate [27, 28]. Both the coverage and accuracy rates can be integrated together through a ratios operation proposed by Saaty [29]. One point to note is that the rank of \( x \) higher than or equal to the rank of \( \overline{x} \) with respect to \( q_j \).

![Fig. 1 A dominating rule for approximations](image1)

![Fig. 2 The structure of criteria interdependence](image2)

### 3.3 The structure of criteria interdependence

The structure of criteria interdependence can be constructed by induction rules of Proposition 3 by considering the endless interdependence among criteria as Fig. 2. Each arrow means a conditional dependence of one criterion on another criterion. Technically, the conditional dependence represents a membership degree of a criterion close to another criterion, i.e., an influence. On the other direction of the influence is the sensitivity of a criterion from another criterion. The interdependence rules can be constructed as the following propositions.

**Proposition 6**: An interdependence rule \([q_{ji,t} \rightarrow q_{ji,t} \land q_{ji,t} \rightarrow q_{ji,t} \rightarrow q_{ji,t} \rightarrow q_{ji,t} ] \equiv R_{ji} \land R_{ik} \rightarrow R_{jk} \)

The rule has approximations \( P_{jk,t} = \{x \in q_{ji,t}, D^+_p(x) \subseteq q_{ji,t}, P = \{q_j, q_k\}, \overline{P}_{jk,t} = U - P_{jk,t} \} \), where \( D^+_p(x) \) performs like \( D^+_p(x) \). Its qualities include \( CR_{jk} = \frac{|P_{jk,t}|}{|q_{ji,t}|}, \quad \alpha_{jk} = \frac{|P_{jk,t}|}{|\overline{P}(q_{ji,t})|} \), and \( g_{jk} = \min \left\{ \max \left\{ CR_{jk} \times \alpha_{jk}, g_{jk} \right\}, g_{jk} \right\} \) which restricts the accurate coverage rate as the minimum possibility along the conjunctives.

**Proposition 7**: The possibility of an interdependence rule is expressed as \( Pos_{jk} = Pos((R_{ji} \cup (R_{ji} \land R_{ik} \rightarrow R_{jk}) \cup ...)) = \max(Pos(R_{ji}), Pos(R_{ji} \land R_{ik} \rightarrow R_{jk}), ...) \)

So, \( Pos_{jk} = \max\left\{ g_{jk}^{r+1}, g_{jk}^{r+1}, g_{jk}^{r+1}, ..., \right\} \) where \( Pos(R_{jk}) = g_{jk}, Pos(R_{jk}) = g_{jk}, \ldots, \) etc.

**Proposition 8**: A conjunctive possibility, \( g_{jk}^{r} \)

\( Pos(R_{ji} \land R_{ik} \rightarrow R_{jk}) \leq \min\{ Pos(R_{ji}), Pos(R_{ik}) \} \). So, \( g_{jk}^{r} \leq \min\{ g_{jk}^{r}, g_{jk}^{r}, g_{jk}^{r}, g_{jk}^{r} \} \) and \( g_{jk}^{r} \leq \min\{ g_{jk}^{r}, g_{jk}^{r}, g_{jk}^{r}, g_{jk}^{r} \} \).

### 3.4 DEMATEL induction

A matrix composed of the accurate coverage rates is designed as Ex. (1) by DEMATEL considering the interdependence between any two criteria iteratively.

\[
G = G' \cup (G' \cdot G') \cup (G' \cdot G' \cdot G') \cup ...
\]  
(1)

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Prove the details and further explanations of the above propositions and equations in the context or related literature. Provide references to support the theoretical aspects.
The network relationship map (NRM) of the criteria interdependence, $G = [g'_{ij}]$, is constructed to show the influence and sensitivity among criteria. The membership degrees of a criterion to the others means if it changes then the others will change. Its sum is called as the influence. The membership degrees from the others to a criterion means if the others change will cause its change. Its sum is called sensitivity. They are formulated as: Influence vectors: $r = (r_1, ..., r_j, ..., r_m)$, $r_j = \sum_{i=1}^{m} g'_{ij}$; Sensitivity vectors: $d = (d_1, ..., d_j, ..., d_m)$,

$$d_j = \sum_{i=1}^{m} (g'_{ij})^T = \sum_{i=1}^{m} g_{ij}$$

where $r_j$ represents the influence of $q_j$ in the others, $d_i$ represents the sensitivity of $q_j$ to the others. NRM is then illustrated with vertical influence and horizontal sensitivity.

As users can see the criteria, $q_8$, $q_{13}$, and $q_{19}$ have biggest influence and $q_{14}$ has the biggest sensitivity. Apparently, the business efficiency is not only highly sensitive but also influencing. For policy making it needs to consider three aspects, i.e., infrastructure, government, and business efficiency.

**Discussion and concluding remarks**

This research finds out the significant criteria of the influence and sensitivity. The resulted Fig. 3 illustrates most criteria have higher influence effects than sensitivity. The institution framework ($q_8$), nosiness finance ($q_{13}$), and management practice ($q_{14}$) appear uniquely important due to its sensitivity and influence. Especially, institution framework ($q_8$) plays a role like the benchmark of competitiveness because it is public and under estimation of WCY. Stakeholders can easily realize the going of nations. Technically, the DEMATEL provides knowledge of criteria interdependence by using the membership degrees to express the influencing and sensitivity for each criterion. Users can easily find out significant criteria in the preferences system.

**REFERENCES**


