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Validating the Analytic Hierarchy/Network Processes

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Summary. This paper contains a number of examples showing that knowledgeable people making judgments using the Analytic Hierarchy Process can match objective measures rather closely. The purpose of doing such validation exercises is to build confidence that our judgments can give good results when objective measures are not available. We consider the following kinds of questions:
What is a relative ratio scale?
How does one translate real-world data into a relative scale so it can be compared to a relative priority scale obtained from the AHP?
How does one validate AHP results against results derived from complex mathematical, physics or economic formulas rather than from a simple linear scale?
How does one measure how close two vectors are when they are relative ratio scale vectors?

Keywords: Validation of the AHP, validation of the ANP, validation of hierarchies, validation of networks, relative ratio scales, incompatibility index

1. Introduction

In this paper we give several examples which show that people, making pairwise comparison judgments using the Fundamental Scale of the AHP can capture reality well. When independent data can be obtained in other ways the AHP results can be validated against it. Since AHP priority vectors are in the form of relative ratio scale numbers, sometimes the data one is validating against are from a set of known measures using a ratio scale such as kilometers or pounds. In this case convert the data into a relative ratio scale by normalizing. This form is then comparable to an AHP vector. The data, however, does not necessarily have to be from a ratio scale from physics. We have done validation exercises by estimating the market share of companies based on subjective elements in an AHP or ANP model, or by estimating the relative number of votes for candidates in a presidential election where the results could be converted to relative ratio scales by normalizing even though the data were votes and were not from physical scales. The form of the original data being used for validation does not matter so long as it can be converted to a relative ratio scale, usually by normalizing, for comparison with an AHP vector.

At other times, rather than normalizing the real world data so it resembles an AHP priority vector, we have applied the AHP priority vector in some creative way to give results in real-world terms such as in the example on the turn-around of the US economy given later. In that exercise we transformed the AHP vector into a single number that was an estimate of how many months until the US economy would turn around. We used expected value computations and treated the AHP vector elements as likelihoods.

There are two ways the AHP can be validated. One is to consider the elements and connections as influences driving an outcome and the alternatives of the model are this outcome which can be compared to some known data from the real world. In this case AHP is used as a predictive tool. The other way is to use the AHP to determine the best alternative to use to reach some desired situation. In this case AHP is used as a decision making tool. It is clear in the first situation whether the prediction of the AHP succeeded – either the results match or not. Failure may, however, be attributable to the way the user sets up the model. In the second situation success is more difficult to establish. Whether or not a decision was successful may not be known for years and is always a matter of interpretation sometimes influenced by events happening that could not be known at the time the decision was made. All of the validation examples here fall into the first category where it was known that they were successful.

2. The Compatibility Index

The compatibility index is a measure for determining how close an estimate using AHP is to the actual relative values you are trying to match. We shall use the well-known area exercise explain the compatibility index. Figure 1 shows five figures. The object is to show that you can estimate the relative sizes of the figures rather closely using judgments.

Create an AHP pairwise comparison matrix by comparing the areas in pairs as shown in Table 1. The AHP relative priority vector which is an estimate of the relative sizes of the areas is obtained by computing the principal eigenvector of this matrix. The actual relative areas are obtained by measuring the figures with a ruler and applying rules from geometry to determine the areas in square units – centimeters, inches, or any other unit. Sum the square units for the five figures and obtain a vector of their relative sizes by normalizing. Regardless of the units used, the area of the circle relative to the square is always the same (about twice as big).

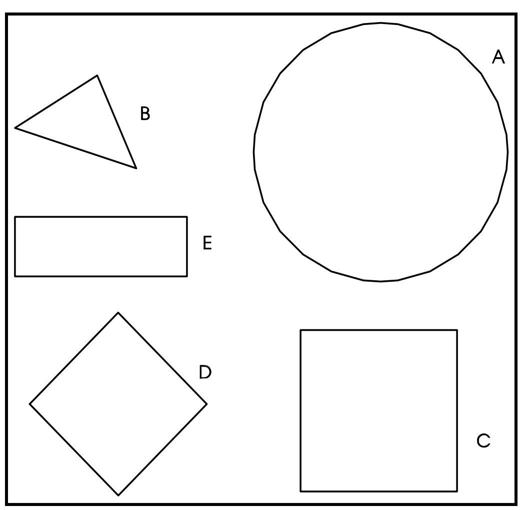


Figure 1. Find the Relative Areas of these Five Figures

Readers can do this exercise using their own judgments. Use the integers from 1 to 9 of the Fundamental Scale of the AHP as well as decimals that lie between them to make the judgments. If you do not have AHP software you can approximate the priority vector by adding across the rows and normalizing the resulting vector.

The values of the AHP priorities and the normalized areas in Table 1 look rather close if the differences are considered. But the AHP is based on ratios, not differences, so it is appropriate to use ratios to measure closeness. The compatibility index given below is a useful ratio-based measure for judging when any two ratio scale vectors are close.

Form the matrix $W = \frac{w_i}{w_j}$ from the AHP priority vector in the next to last column of Table 1. Form the matrix $X = \frac{x_i}{x_j}$ from the vector of normalized actual data in the last column of Table 1. The transpose of X is $X^T = \frac{x_j}{x_i}$. Form the Hadamard product $W \times X^T$ by multiplying the corresponding elements in the two matrices. When dealing with Hadamard products either matrix may be used as the transpose matrix. To get the result sum all the elements of the new matrix and divide by n^2. If the two vectors are identical this yields a value of 1. The farther the result is from 1 the more different the vectors.

Table 1. Pairwise Comparisons of the Five Geometric Figures

Areas	A	B	C	D	E	AHP Priority Vector	Actual Areas (Normal-ized)

A	1	9	2.5	3.5	5	0.489958	0.471
B	1/9	1	1/5	1/2.5	1/2	0.050038	0.050
C	1/2.5	5	1	2	2.5	0.234992	0.234
D	1/3.5	2.5	1/2	1	1.5	0.130579	0.149
E	1/5	2	1/2.5	1/1.5	1	0.094433	0.096

Consistency Ratio = 0.0033 (for the judgment matrix - not to be confused with the Compatibility Index)
Compatibility Index = 1.003426

To compute the Compatibility Index that shows how close the AHP Priority Vector is to the normalized vector of the actual areas first form the pairwise comparison matrix W from the AHP Priority Vector in Table 1, shown below:

$$W = \begin{bmatrix} 1.000000 & 9.791718 & 2.084999 & 3.752196 & 5.188419 \\ 0.102127 & 1.000000 & 0.212935 & 0.383201 & 0.529878 \\ 0.479617 & 4.696271 & 1.000000 & 1.799616 & 2.488452 \\ 0.266511 & 2.609597 & 0.555674 & 1.000000 & 1.382769 \\ 0.192737 & 1.887226 & 0.401856 & 0.723187 & 1.000000 \end{bmatrix}$$

Form the pairwise comparison matrix X from the vector of the actual areas (normalized) from Table 1. Take its transpose and multiply by the matrix W as shown below.

$$X = \begin{bmatrix} 1.000000 & 9.420000 & 2.012821 & 3.161074 & 4.906250 \\ 0.106157 & 1.000000 & 0.213675 & 0.335570 & 0.520833 \\ 0.496815 & 4.680000 & 1.000000 & 1.570470 & 2.437500 \\ 0.316348 & 2.980000 & 0.636752 & 1.000000 & 1.552083 \\ 0.203822 & 1.920000 & 0.410256 & 0.644295 & 1.000000 \end{bmatrix}$$

$$X^T = \begin{bmatrix} 1.000000 & 0.106157 & 0.496815 & 0.316348 & 0.203822 \\ 9.420000 & 1.000000 & 4.680000 & 2.980000 & 1.920000 \\ 2.012821 & 0.213675 & 1.000000 & 0.636752 & 0.410256 \\ 3.161074 & 0.335570 & 1.570470 & 1.000000 & 0.644295 \\ 4.906250 & 0.520833 & 2.437500 & 1.552083 & 1.000000 \end{bmatrix}$$

$$W \times X^T = \begin{bmatrix} 1.000000 & 1.039461 & 1.035859 & 1.187000 & 1.057512 \\ 0.962037 & 1.000000 & 0.996535 & 1.141939 & 1.017366 \\ 0.965382 & 1.003477 & 1.000000 & 1.145909 & 1.020903 \\ 0.842460 & 0.875704 & 0.872670 & 1.000000 & 0.890911 \\ 0.945616 & 0.982930 & 0.979525 & 1.122446 & 1.000000 \end{bmatrix} \quad \begin{array}{r} \\ \\ \\ \\ \\ \\ \text{Total} \end{array} \begin{bmatrix} \textit{RowSums} \\ 5.319832 \\ 5.117878 \\ 5.135671 \\ 4.481744 \\ 5.030516 \\ 25.0856424 \end{bmatrix}$$

Divide the total, the sum of all the elements in the matrix by n^2 or 25 to get the compatibility index. The Compatibility Index thus obtained is $\dfrac{25.0856424}{25} = 1.003426$

The nearer the computed value is to 1, the nearer the two vectors being compared, so by this measure these two vectors are close.

A relative priority vector depends on the set of elements being pairwise compared. It would be different if a sixth area were introduced. But the ratios of the elements obtained from the original priority vector would, however, remain the same in the new priority vector with 6 elements: the circle, for example, should always be about twice the square (to within judgmental error), regardless of how many additional areas are added.

3. Some Validation Examples

We shall now give several validation examples that have appeared before in scattered locations throughout the literature. We felt it would be useful to collect these experiments that have been done over the years m in one place as we have become more and more convinced that the validation of the AHP/ANP by these types of exercises is extremely important in promoting the use of the theory. A second reason to collect these examples is to show the wide variety of scales against which the validation exercises were being performed: areas in square units, inverses of distances squared, volume units of liquids, protein units, weight in pounds, kilowatts, GNP units – a financial measure, number of games won, the exchange rate of the US dollar versus the Japanese Yen, the average number of children in a family in India, the outcome of a vote in the US Congress, time in months to a turnaround of the US economy, and relative market share in the cereal industry. The market share validation exercise using a network model with feedback has been done over and over for many industries ranging from the sport shoe industry to the pizza industry to the mass marketer industry (Walmart and other such chains) and many others. The point of all these examples is to show the rich variety of validations of the AHP that have been carried out.

The first ten examples are hierarchical models. We then turn to network models for the last three examples.

3.1 Optics Example

Four identical chairs were placed on a line from a light source at distances of 9, 15, 21, and 28 yards respectively. The chairs had leather backs that while not shiny reflected the light rather well. We assume that our perception of the light we see being reflected from the chair is about what the brightness of light is at the point where the chair is located. The question is: "Can we use judgment to determine the brightness of light on the four chairs?" The experiment consisted of the observer standing by the light, looking at the chairs and making pairwise comparison judgments as to the relative brightness of light he sees. The priority vector obtained from the AHP judgment matrix should give the relative brightness of light at the four chairs. Fortunately, to validate our AHP results, we have the inverse square law of optics which states that the brightness of light varies inversely as the square of the distance from the source to tell us what the brightness of the light should be at each chair. If we use the formula $\frac{1}{d^2}$ to compute the light on the closest chair first, then on the second chair and divide the two numbers we obtain an absolute number indicating how many times brighter the first chair appears than the second, and continue the process for the third and fourth chairs. This gives us a relative absolute vector having four elements for the brightness of light on the four chairs. Closer chairs would of course be brighter and have higher values. Since we know the distance d of each chair from the light source we can compute the brightness of light for each using this law. The farther the chair from the light, the less bright it will appear. Normalize these four numbers for brightness to yield a vector of relative brightness for the four chairs as shown in the last column of

Table 2.

The judges for the first matrix were the author's young children, ages 5 and 7 at that time, who gave qualitative consensus judgments, that is, they expressed their opinion using the verbal equivalents of the numbers from the AHP Fundamental Scale. The judge for the second matrix was the author's wife, who was not present when the children were giving their judgments. The judgment matrices and their corresponding priority vectors are:

Relative visual brightness (1st Trial)

	C_1	C_2	C_3	C_4
C_1	1	5	6	7
C_2	1/5	1	4	6
C_3	1/6	1/4	1	4
C_4	1/7	1/6	1/4	1

Relative visual brightness (2nd Trial)

	C_1	C_2	C_3	C_4
C_1	1	4	6	7
C_2	1/4	1	3	4
C_3	1/6	1/3	1	2
C_4	1/7	1/4	1/2	1

Relative brightness eigenvector (1st Trial)

0.61
0.24
0.10
0.05

$\lambda_{max} = 4.39$, C.R.= 0.14

Relative brightness eigenvector (2nd Trial)

0.62
0.22
0.10
0.06

$\lambda_{max} = 4.10$, C.R.= 0.03

Table 2. Brightness of Light as Predicted by the Inverse Square Law of Optics

	d	d^2	$1/d^2$	$(1/d^2)$/Sum
Chair 1	9	81	0.01234568	0.607168
Chair 2	15	225	0.00444444	0.218581
Chair 3	21	441	0.00226757	0.111521
Chair 4	28	784	0.00127551	0.062730
		Sum	0.02033321	1.000000

To the surprise and delight of Thomas L. Saaty who was conducting this type of validation experiment for the first time it turned out that the relative brightness of light from both trials was quite close to that predicted by the inverse square law of optics. The results for the first and second trials can be compared with the last column of

Table 2 which was calculated using the inverse square law of optics. It is interesting and important to observe that the judgments here have captured what a law of physics predicts should be the result. This should give us some confidence in the process and in our ability to make judgments. It would seem that our judgments work well in this situation we should be able to capture reality in other areas of perception or thought as well.

The relative brightness of light was gotten by direct observation. The distances of the chairs from the light were not known to the people giving the judgments and were not involved in the calculations of the results from the AHP. This is typical of AHP results. AHP relative priority vectors are obtained directly from judgments while physics formulas always involve making intermediate measurements using some kind of standardized physical measuring device that are then manipulated through the use of formulas.

To change the Optics law results into a vector that can be compared against the vectors of Trials 1 and 2 one first calculates the brightness of light given by the law, then changes it to a relative absolute vector by summing the results for the 4 chairs and dividing each by this sum. Note that the results from the Optics law may be quite sensitive so it requires great care in measuring the distances; if the first object is very close to the light source it would then absorb most of the value of the relative vector and a small error in its distance from the source would yield great error in the other values. What is noteworthy from this sensory experiment is that the validation goes both ways. The observation or hypothesis of the Optics Law that the observed intensity of illumination varies inversely with the square of the distance is validated with the results from the AHP experiment as well as vice versa. The more carefully designed any experiment that tests natural law (or hypothesis, which is what all natural laws are in the beginning), the better the results that will be obtained.

The Compatibility Index for the Optics Law versus Trial 1 is 1.014573 and versus Trial 2 is 1.002595. Thus Trial 2 is somewhat closer to what the formula predicts.

3.2 Relative Consumption of Drinks

Table 3 shows how an audience of about 30 people, using consensus to arrive at each judgment, estimated the relative consumption of drinks in the United States (which drink is consumed more in the US and how much more than another drink?). The derived vector of relative consumption and the actual vector, obtained by normalizing the consumption given in official statistical data sources, are at the bottom of the table.

Table 3. Relative Consumption of Drinks

Which Drink is Consumed More in the U.S.?

An Example of Estimation Using Judgments

Drink Consumption in the U.S.	Coffee	Wine	Tea	Beer	Sodas	Milk	Water
Coffee	1	9	5	2	1	1	1/2
Wine	1/9	1	1/3	1/9	1/9	1/9	1/9
Tea	1/5	2	1	1/3	1/4	1/3	1/9
Beer	1/2	9	3	1	1/2	1	1/3
Sodas	1	9	4	2	1	2	1/2
Milk	1	9	3	1	1/2	1	1/3
Water	2	9	9	3	2	3	1

The derived scale based on the judgments in the matrix is:

Coffee	Wine	Tea	Beer	Sodas	Milk	Water
.177	.019	.042	.116	.190	.129	.327

with a consistency ratio of .022.

The actual consumption (from statistical sources) is:

.180	.010	.040	.120	.180	.140	.330

The Compatibility Index is 1.055.

3.3 Relative Amount of Protein in Seven Foods

An exercise to determine the relative amount of protein in seven foods was done by a group of people using consensus judgments. The judgments and results are given in

Table 4. In this exercise the apple has no protein so it should receive a zero. But leaving the zero there would cause division by zero and the Compatibility Index computations would blow up. In this case, one should remove the element with zero weight and re-calculate without it; that is, re-normalize the AHP derived scale as shown in

Table 4.

Table 4. Which Food has more Protein?

Protein in Food	A	B	C	D	E	F	G
A: Steak	1	9	9	6	4	5	1
B: Potatoes		1	1	1/2	1/4	1/3	1/4
C: Apples			1	1/3	1/3	1/5	1/9
D: Soybean				1	1/2	1	1/6
E: Whole Wheat Bread					1	3	1/3
F: Tasty Cake						1	1/5
G: Fish							1

The derived scale and actual values are:

Steak	Potatoes	Apples	Soybean	W. Bread	T. Cake	Fish
.345	.031	.030	.065	.124	.078	.328
.370	.040	.000	.070	.110	.090	.320

with a consistency ratio of .028.

Table 5. Protein Results Recomputed with the Apple Removed.

	Steak	Potatoes	Soybean	W. Bread	T. Cake	Fish
AHP	.355	.032	.067	.128	.080	.338
Actual	.370	.040	.070	.110	.090	.320

Compatibility Index 1.0142873.

3.4 Relative Weights of Objects

The matrix in

Table 6 gives the pairwise comparison judgments for the weights of five objects by a judge who lifted two objects at a time using both hands. The actual weights were found later by weighing the objects on a scale, and their relative weights computed. The two vectors appear to be very close according to the differences between their elements. The Compatibility Index is shown below the table.

Table 6. Pairwise Comparisons of the Weights of Five Objects

Weight	Radio	Type-writer	Large Attaché Case	Project-or	Small Attaché Case	Eigen-vector	Actual Rela-tive Weight
Radio	1	1/5	1/3	1/4	4	.09	.10
Type-writer	5	1	2	2	8	.40	.39
Large Attaché	3	1/2	1	1/2	4	.18	.20
Projector	4	1/2	2	1	7	.29	.27
Small Attaché	1/4	1/8	1/4	1/7	1	.04	.04

Compatibility Index 1.00507987

3.5 Relative Electric Consumption of Household Appliances

In Table 7 we give the matrix of paired comparison judgments estimating the consumption of electricity of common household appliances. The estimates (by consensus) were given by students in Electrical Engineering. The actual relative weights were computed later using known kilowatt hour consumption for the appliances.

Table 7. Household Appliances Relative Electricity Consumption (Kilowatt Hours)

Annual Electricity	Elec-tric Range	Refrig	TV	Dish-Wash	Iron	Hair Dryer	Radio	AHP Eigen-vector	*Actual Relative Weights*
Elec.Range	1	2	5	8	7	9	9	.393	*.392*
Refrig.	1/2	1	4	5	5	7	9	.261	*.242*
TV	1/5	1/4	1	2	5	6	8	.131	*.167*
Dishwash.	1/8	1/5	1/2	1	4	9	9	.110	*.120*
Iron	1/7	1/5	1/5	1/4	1	5	9	.061	*.047*
Hair-dryer	1/9	1/7	1/6	1/9	1/5	1	5	.028	*.028*

Radio	*1/9*	*1/9*	*1/8*	*1/9*	*1/9*	*1/5*	*1*	*.016*	*.003*

Compatibility Index 1.455.

This Compatibility Index is quite high. Again the problem is that one of the elements does not belong in the group. The electricity consumption by the radio is so small in comparison with the others that the group is not homogeneous; that is, it does not satisfy the requirement that no element should be more than 9 times another. For a better gauge of relative consumption remove the radio from both the AHP eigenvector and the actual results vector and renormalize as we did before in the example of estimating protein in foods. The results with the radio removed are shown in Table 8.

Table 8. Electric Appliance Power Consumption with Radio Removed.

	Electric Range	Refriger-ator	TV	Dish-washer	Iron	Hairdryer
AHP	.399	.265	.133	.112	.062	.028
Actual	.394	.243	.168	.120	.047	.028

Compatibility Index is now 1.0240673

3.6 Relative Wealth of Seven Nations

Very early in the history of the AHP, T. Saaty and M. Khouja, did the following exercise while traveling on an airplane in 1973 and reported on it in "A Measure of World Influence," *Journal of Peace Science*, Spring, 1976. They simply used their common knowledge about the relative influence and standing of these countries in the world and without referring to any specific economic data related to GNP values. The two results are close and demonstrate that the general understanding an interested person has about a problem can be used to advantage to make fairly good estimates through paired comparisons. The eigenvector solution of the AHP paired comparison matrix of the estimated relative wealth of countries is shown in the first column of Table 11 and appears to be quite close to the normalized GNP values in the last column.

Table 9 gives the judgments using the AHP 1-9 scale and Table 10 gives the priorities derived from AHP and the actual and relative GNP values.

The eigenvector solution of the AHP paired comparison matrix of the estimated relative wealth of countries is shown in the first column of Table 11 and appears to be quite close to the normalized GNP values in the last column.

Table 9. Paired Comparisons of the Relative Dominance in Wealth of Seven Nations

$$\begin{pmatrix} & U.S & U.S.S.R & China & France & U.K & Japan & W.Germany \\ U.S & 1 & 4 & 9 & 6 & 6 & 5 & 5 \\ U.S.S.R & 1/4 & 1 & 7 & 5 & 5 & 3 & 4 \\ China & 1/9 & 1/7 & 1 & 1/5 & 1/5 & 1/7 & 1/5 \\ France & 1/6 & 1/5 & 5 & 1 & 1 & 1/3 & 1/3 \\ U.K & 1/6 & 1/5 & 5 & 1 & 1 & 1/3 & 1/3 \\ Japan & 1/5 & 1/3 & 7 & 3 & 3 & 1 & 2 \\ W.Germany & 1/5 & 1/4 & 5 & 3 & 3 & 1/2 & 1 \end{pmatrix}$$

Compatibility Index = 1.07978837

Table 10. The Outcome of Estimated Relative Wealth Versus the Actual GNP

	Estimated Relative Wealth	Actual GNP (1972)	Normalized GNP Values
U.S	.427	1,167	.413
U.S.S.R	.23	635	.225
China	.021	120	.043

France	.052	196	.069
U.K	.052	154	.055
Japan	.123	294	.104
W. Germany	.094	257	.091

3.7 World Chess Championship Outcome Validation – Karpov-Korchnoi Match

The hierarchy shown in Figure 2 was used to predict the outcome of a world chess championship series of matches between Karpov and Korchnoi. The judgments were from ten grandmasters in the then Soviet Union and the United States who responded to questionnaires that were mailed to them. The predicted outcomes included the number of games played, drawn and won by each player, and the predictions either came out exactly as the matches turned out later or adequately close to predict the winner of the entire series. The predicted winner of the match, Karpov by 6 to 5 games over Korchnoi, was notarized before the match took place. The paper was later mailed to the editor of the *Journal of Behavioral Sciences* 1980, along with the notarized statement about who the winner would be and by how much. For more details see the co-authored book by Saaty and Vargas: *Prediction, Projection and Forecasting*, Kluwer, 1991. The factors that play into winning a chess match used as the criteria in the hierarchy were defined in Table 11.

Table 11. Definitions of Chess Factors

T (1) Calculation (Q): The ability of a player to evaluate different alternatives or strategies in light of prevailing situations.
B (2) Ego (E): The image a player has of himself as to his general abilities and qualification and his desire to win.
T (3) Experience (EX): A composite of the versatility of opponents faced before, the strength of the tournaments participated in, and the time of exposure to a rich variety of chess players.
B (4) Gamesmanship (G): The capability of a player to influence his opponent's game by destroying his concentration and self-confidence.
T (5) Good Health (GH): Physical and mental strength to withstand pressure and provide endurance.
B (6) Good Nerves and Will to Win (GN): The attitude of steadfastness that ensures a player's health perspective while the going gets tough. He keeps in mind that the situation involves two people and that if he holds out the tide may go in his favor.
T (7) Imagination (IW: Ability to perceive and improvise good tactics and strategies.
T (8) Intuition (IN): Ability to guess the opponent's intentions.
T (9) Game Aggressiveness (GA): The ability to exploit the opponent's weaknesses and mistakes to one's advantage. This is occasionally referred to as "killer instinct."

T (10) Long Range Planning (LRP): The ability of a player to foresee the outcome of a certain move, set up desired situations that are more favorable, and work to alter the outcome.
T (1 1) Memory M: Ability to remember previous games.
B (12) Personality (P): Manners and emotional strength, and their effects on the opponent in playing the game and on the player in keeping his wits.
T (13) Preparation (PR): Study and review of previous games and ideas.
T (14) Quickness (Q): The ability of a player to see clearly the heart of a complex problem.
T (15) Relative Youth (RY): The vigor, aggressiveness, and daring to try new ideas and situations, a quality usually attributed to young age.
T (16) Seconds (S): The ability of other experts to help one to analyze strategies between games.
B (17) Stamina (ST): Physical and psychological ability of a player to endure fatigue and pressure.
T (18) Technique M: Ability to use and respond to different openings, improvise middle game tactics, and steer the game to a familiar ground to one's advantage.

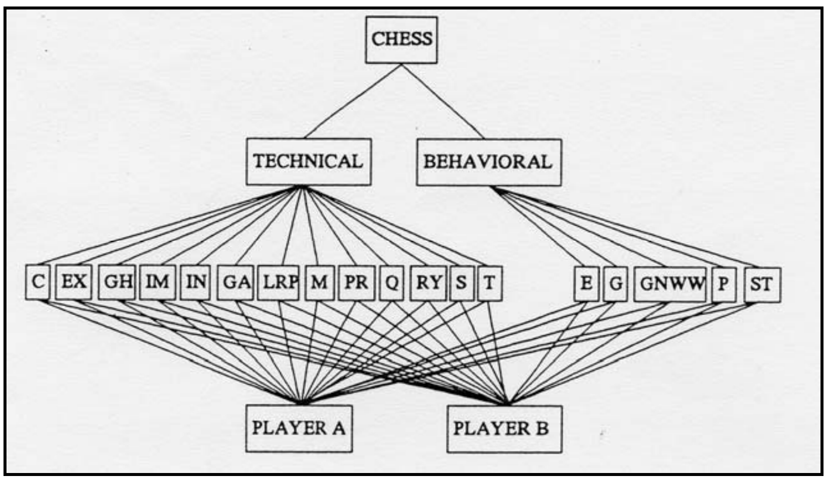


Figure 2. Hierarchy for Predicting Chess Winner

3.8 Predicting the Monetary Exchange Rate for the Dollar versus the Yen

In the late 1980's three economists at the University of Pittsburgh, Professors A. Blair, R. Nachtmann, and J. Olson, worked with Thomas Saaty on predicting the yen/dollar exchange rate. The paper was published in *Socio-Economic Planning Sciences* 31, 6(1987). The predicted value was fairly close to the average exchange rate for the yen for a considerable number of months after that. Figure 3 shows how the decision was structured as a hierarchy and gives the outcome.

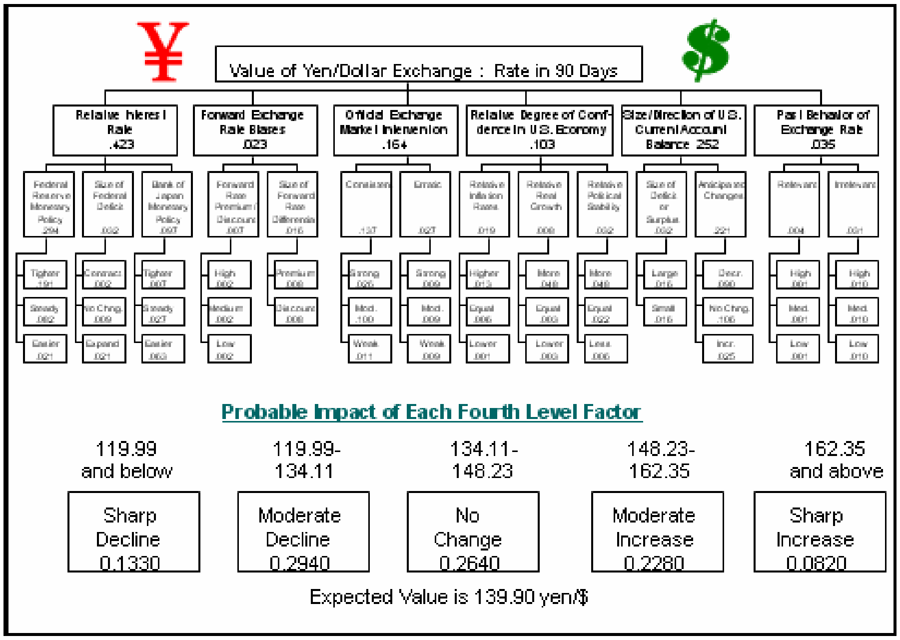


Figure 3. Yen/Dollar Exchange Rate

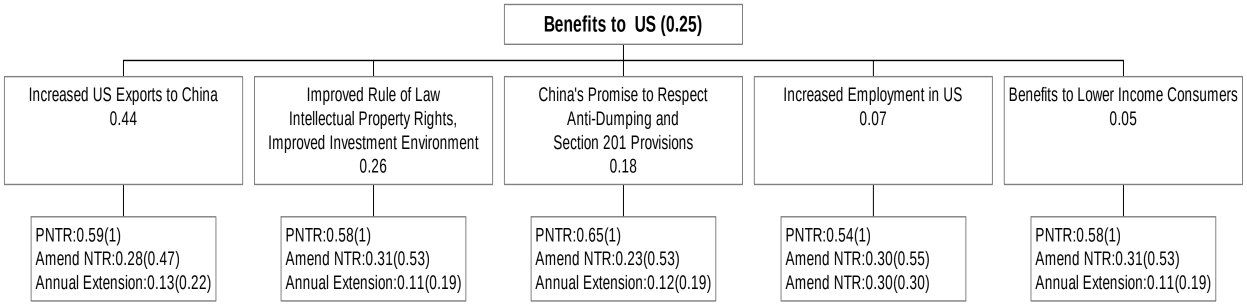
3.9 Estimating the Number of Children in Rural Indian Families

In a hierarchy with the goal of estimating the optimal family size in India (from a study published T. L. Saaty and Molly Wong in the *Journal of Mathematical Sociology*, 1983, Vol.9 pp. 181-209), there were four major criteria: Culture (with subcriteria: Religion, Women Status, Manhood), Economic factors (with subcriteria: Cost of Child Rearing, Old Age Security, Labor, Economic Improvement, Prestige and Strength), Demographic factors (with subcriteria: Short Life Expectancy, High Infant Mortality) and the Availability and Acceptance of Contraception (with subcriteria: High Level of Availability and Acceptance of Contraception, Medium level of Availability and Acceptance of Contraception, Low Level of Availability and Acceptance of Contraception. At the bottom three alternatives were considered: Families with 3 or Less Children, Families with 4 to 7 Children, and Families with 8 or More Children. The outcome of this example for reasons explained in the research paper had two projections of 5.6 and 6.5 children per family (due to regional differences.) The actual value we obtained from the literature after the study was done was that there were 6.8 births per woman in 1972 and 5.6 in 1978.

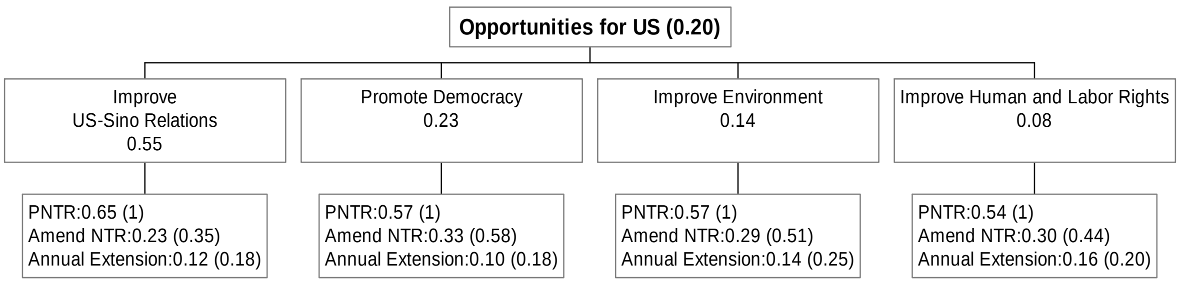
3.10 Decision by the US Congress on China Joining The World Trade Organization

This study was done in 1999 by Thomas L. Saaty and Min Cho prior to the US Congress voting on the issue of China joining the World Trade Organization (WTO). Briefly, the alternatives of the decision are:
1 - Passage of a clean PNTR bill: Congress grants China Permanent Normal Trade Relations (PNTR) status with no conditions attached. This option would allow implementation of the November 1999 WTO trade deal between China and the Clinton administration. China would also carry out other WTO principles and trade conditions.
2 - Amendment of the current NTR status bill: This option would give China the same trade position as other countries and disassociate trade from other issues. As a supplement, a separate bill may be enacted to address other matters, such as human rights, labor rights, and environmental issues.
3 - Annual Extension of NTR status: Congress extends China's Normal Trade Relations (NTR) status for one more year, and, thus, maintains the status quo.

Four hierarchies were considered, the benefits and opportunities hierarchies shown in Figure 4 and the costs and risks hierarchies shown in Figure 5. The outcomes were combined to derive the final priorities for how Congress was going to vote. and in fact China was later admitted to the WTO. Figure A2-5 and Tables A2-10 and A2-11 summarize the results.

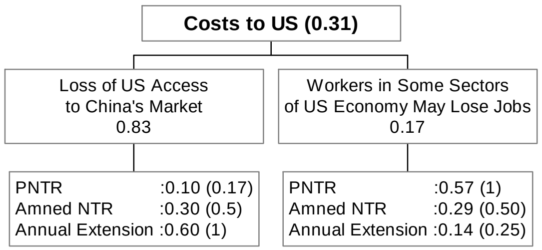


Benefits Synthesis (Ideal): PNTR 1.00, Amend NTR 0.51, Annual Extension 0.21

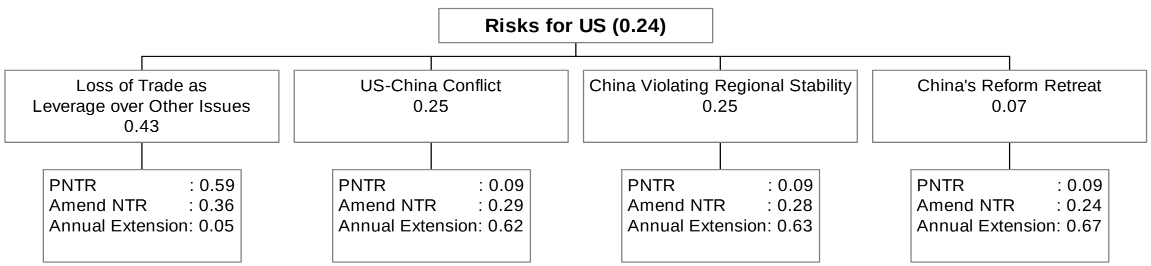


Opportunities Synthesis (Ideal): PNTR 1, Amend NTR 0.43, Annual Extension 0.13

Figure 4 Benefits and Opportunities Hierarchies



Costs Synthesis (which is more costly, Ideal): PNTR 0.31, Amend NTR 0.50, Annual Extension 0.87



Risks Synthesis (which is more risky, Ideal): PNTR 0.54, Amend NTR 0.53, Annual Extension 0.58

Figure 5. Costs and Risks Hierarchies

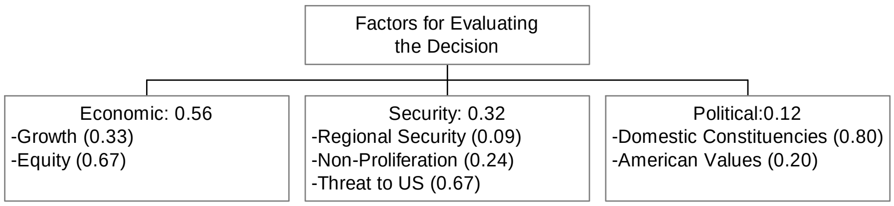


Figure 6. Prioritizing Strategic Criteria to be used in Rating the BOCR

How to derive the priority shown next to the goal of each of the four hierarchies in Figure 5 is outlined in the table below. We rated each of the four merits: benefits, costs, opportunities and risks of the dominant PNTR alternative, as it happens to be in this case, in terms of intensities for each assessment criterion. The intensities, Very High, High, Medium, Low, and Very Low were themselves prioritized in the usual pairwise comparison matrix to determine their priorities. We then assigned the appropriate intensity for each merit on all assessment criteria using the priorities listed above the table in the computations of the BOCR priorities found in the bottom row of Table 12. The computations are performed for benefits, for example, by multiplying the criterion value times the subcriterion value times the intensity priority and summing down the benefits column.

Table 12. Priority Ratings for the Merits: Benefits, Costs, Opportunities, and Risks
Intensities and priorities: Very High (0.42), High (0.26), Medium (0.16), Low (0.1), Very Low (0.06)

Criteria	*Subcriteria*	Benefits	Opportunities	Costs	Risks
Economic (0.56)	Growth (0.19)	High	Medium	Very Low	Very Low
	Equity (0.37)	Medium	Low	High	Low
Security (0.32)	Regional (0.03)	Low	Medium	Medium	High
	Non-Proliferation (0.08)	Medium	High	Medium	High
	Threat to US (0.21)	High	High	Very High	Very High
Political (0.12)	Constituencies (0.1)	High	Medium	Very High	High
	American Values (0.02)	Very Low	Low	Low	Med-ium
BOCR Priorities		0.25	0.20	0.31	0.24

We are now able to obtain the overall priorities of the three major decision alternatives listed earlier, given as columns in the table below which gives three ways of synthesize for the ideal mode, we see that PNTR (in bold) is the dominant alternative any way we synthesize in the last two columns of Table 13. The two formulas for combining the results of the BOCR are:

$$BO/CR$$

or

$$bB + oO - cC - rR$$

The first formula is analogous to the marginal utility formula in economics and the second formula gives the best results over the long term. When one is trying to validate results when the measures are given in dollars, the second formula is the best one to use because of the capability of subtracting.

Table 13. Two Methods of Synthesizing the BOCR

Alternatives	Benefits (0.25)	Opportunities (0.20)	Costs (0.31)	Risks (0.24)	BO/CR	bB + oO - cC - rR
PNTR	1	1	0.31	0.54	5.97	0.22
Amend NTR	0.51	0.43	0.50	0.53	0.83	-0.07
Annual Exten.	0.21	0.13	0.87	0.58	0.05	-0.33

3.11 Turn Around of the U.S Economy in 2001

We now turn to validation examples using network models.

Let us consider the problem of the turn around of the US economy and introduce 3, 6, 12, 24 month time periods at the bottom (see Blair, Nachtmann, Saaty, and Whitaker, *Socio-Economic Planning Sciences*, 36, 2002, pp. 77-91) Decomposing the problem hierarchically, the top level consists of the primary factors that represent the forces or major influences driving the economy: “Aggregate Demand” factors, “Aggregate Supply” factors, and “Geopolitical Context.” Each of these primary categories was then decomposed into sub-factors represented in the second level. Under Aggregate Demand, we identified consumer spending, exports, business capital investment, shifts in consumer and business investment confidence, fiscal policy, monetary policy, and expectations with regard to such questions as the future course of inflation, monetary policy and fiscal policy. We make a distinction between consumer and business investment confidence shifts and the formation of expectations regarding future economic developments.

Under Aggregate Supply, we identified labor costs (driven by changes in such underlying factors as labor productivity and real wages), natural resource costs (e.g., energy costs), and expectations regarding such costs in the future. With regard to Geopolitical Context, we identified the likelihood of changes in major international political relationships and major international economic relationships as the principal sub-factors. With regard to the sub-factors under Aggregate Demand and Aggregate Supply, we recognized that they are, in some instances, interdependent. For example, a lowering of interest rates as the result of a monetary policy decision by the Federal Reserve should induce portfolio rebalancing throughout the economy. In turn, this should reduce the cost of capital to firms and stimulate investment, and simultaneously reduce financial costs to households and increase their disposable incomes. Any resulting increase in disposable income stimulates consumption and, at the margin, has a positive impact on employment and GNP. This assumes that the linkages of the economy are in place and are well understood. This is what the conventional macroeconomic conceptual models are designed to convey.

The third level of the hierarchy consists of the alternate time periods in which the resurgence might occur as of April 7, 2001: within three months, within six months, within twelve months, and within twenty-four months. Because the primary factors and associated sub-factors are time-dependent, their relative importance had to be established in terms of each of the four alternative time periods. Thus, instead of establishing a single goal as one does for a conventional hierarchy, we used the bottom level time periods to compare the two factors at the top. This entailed creation of a feedback hierarchy known as a "holarchy" in which the priorities of the elements at the top level are determined in terms of the elements at the bottom level, thus creating a loop. Figure 7 shows the holarchy we used to forecast the timing of the economic resurgence. A holarchy is a specialized form of a network.

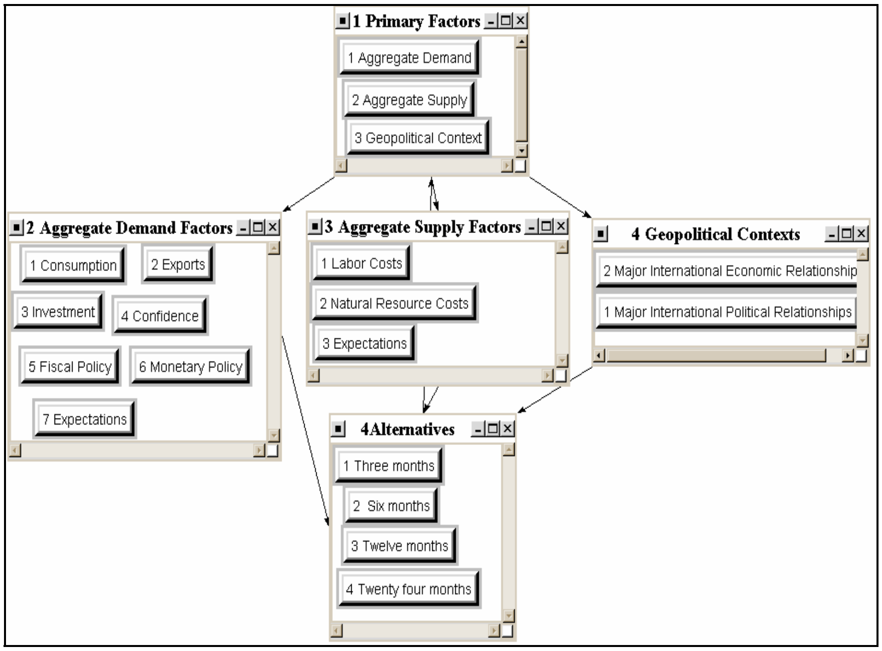


Figure 7. Overall View of the Model, a Holarchy, for Year 2001.

To obtain our forecast, we subsequently multiplied each priority by the midpoint of its corresponding time interval and added the results (as one does when computing expected values) to obtain the results in terms of months as shown in Table 14. These are interpreted as the expected number of months until the turnaround will occur.

We interpreted this to mean that the recovery should occur 8.54 months from the time of the forecasting exercise in April, or in the fall. The Wall Street Journal of July 18, 2003, more than two years after the exercise had the final word on the turnaround date as shown in Figure 8.

Table 14. Summary of Results of the Forecast Time to Turnaround of Economy

Time Period	Midpoint of Time Period	Priority of Time Period	Midpoint x Priority= expected number of months until turnaround
	(Expressed in months with April as 0.)		
Three months	0 + (3 – 0)/2 = 1.5	0.30581	0.45871
Six months	3 + (6 – 3)/2 = 4.5	0.20583	0.92623
Twelve months	6 + (12 – 6)/2 = 9.0	0.18181	1.63629
Twenty-four months	12 + (24 – 12)/2 = 18.0	0.30656	5.51808
TOTAL			8.53932

The Wall Street Journal
Friday, July 18, 2003

Despite Job Losses, the Recession Is Finally Declared Officially Over

JON E. HILSENRATH

The National Bureau of Economic Research said the U.S. economic recession that began in March 2001 ended eight months later, not long after the Sept. 11 terrorist attacks.

Most economists concluded more than a year ago that the recession ended in late 2001. But yesterday's declaration by the NBER-a private, nonprofit economic research group that is considered the official arbiter of recession timing-came after a lengthy internal debate over whether there can be an economic recovery if the labor market continues to contract. The bureau's answer: a decisive yes.

When calling the end to a recession, the NBER focuses heavily on two economic indicators: the level of employment and gross domestic product, or the total value of the nation's goods and services. Since the fourth quarter of 2001, GDP has expanded slowly but consistently-rising 4% through March of 2003.

Employers, however, have eliminated 938,000 payroll jobs since November 2001. In addition, 150,000 people have dropped out of the labor force because they are discouraged about their job prospects, according to the government.

Figure 8. Wall Street Journal Article of 2003 on 2001 Economic Turnaround

3.12 Market Share in the Cereal Industry

The following is one of numerous validation examples done by the author's graduate students in business most of whom work at some company. Many of the examples are done in class in about one hour and without access to data. The answer is only found later on the Internet. The example below was developed by Stephanie Gier and Florian John in March 2002. They write: To become familiar with the SuperDecisions software we have chosen to estimate the market shares for the Ready-to Eat breakfast cereal industry. This idea was born after and delicious breakfast with Post's OREO O's. To see how good our assumptions were, we compare our calculated results with the market shares of 2001. First we created the model. We identified 6 major competitors in the ready to eat cereal market, Kellogg, General Mills, Post, Quaker, Nabisco and Ralston as our alternatives. There were more companies in this market having an actual cumulative market share of roughly about 6% that it turned out later that we had left out. Since we were only concerned with deriving relative values, the relative shares of other residual companies do not matter.

Major impacts on the companies' market shares are:
Price of the products offered (named cost for the consumer)
Advertising / Sales Ratio (how much money is spend for advertising)
Shelf Space (places where the products are located in the stores)
Tools (Selling Tools used to increase sales and market shares)
Distribution/Availability (major distribution channels used to sell product)

These five major impacts (clusters) are further divided as follows:
Tools: (Coupons, trade dealing, in-pack premiums, vitamin fortifications)
Distribution: (Supermarket Chains, Food Stores, Mass Merchandiser)
Shelf Space: (Premium Space, Normal Space, Bad Space)
Cost: (Expensive, Normal, Cheap)
Advertising: (<15%,<14%,<13%,<12%,<11%,<5%)

Their interactions are depicted in Figure 9. Second comparisons were made along with calculations to obtain the final result in Table 15 which compares the outcome with the normalized actual values. Third we compared our calculated market shares with the real market shares for 2001. Table 15 that follows lists estimated market share values and the actual ones taken from the website of the International Data Corporation.

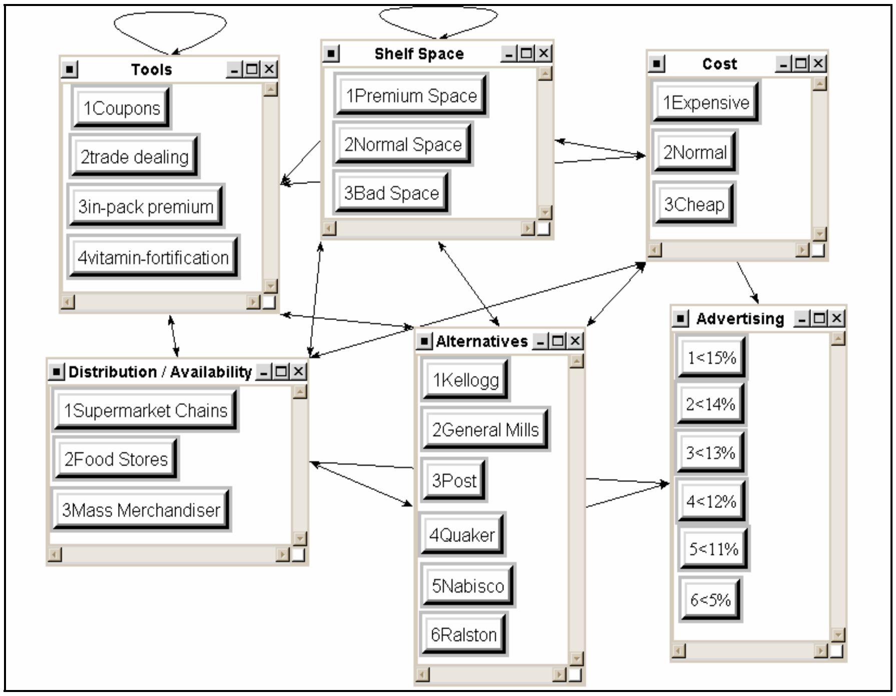


Figure 9. Cereal Industry Market Share

Table 15. Overall Results, Estimated and Actual

Alternatives	Kellogg	General Mills	Post	Quaker	Nabisco	Ralston
Estimated	0.324	0.255	0.147	0.116	0.071	0.087
Actual	0.342	0.253	0.154	0.121	0.057	0.073

Compatibility Index = 1.01403

The compatibility index in this example shows the two vectors are close.