# AHP APPLICATION FOR AIDING OF DECISION-MAKING IN MUNICIPAL ECONOMY

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**Summary:** Municipal economy problems are of complex nature and they require because of various factors. Thus to include them effectively, multicriteria analysis should be applied. Problems of heat production and delivery systems belong to group of such multidisciplinary problems. They were usually resolved in the past using classical numerical methodology that took into account only technical and economical merits. But to obtain more realistic results, that would allow making more effective decisions, special multicriteria tool is required. The Analytical Hierarchy Process seems to be good alternative to fill the existing gap between realms of decision-making and traditionally utilized methodology for resolving problems of heat production and delivery systems. Thanks to simplicity and effectiveness of AHP methodology it is possible to efficiently prioritize quality of applicable alternatives of heat source for the system delivering heat energy to houses in medium sized city located in Poland.

### 1. Introduction

Real technical systems are of complex nature. It arises mainly from not only technology, but also from their multidisciplinary nature. Therefore to effectively make decisions pertaining to choice of solution from limited, but very differentiated set of available alternatives, the multicriteria approach is required.

To evaluate value of existing alternatives different methodologies could be applied e.g. multi-attribute utility technique (MAUT) (Keeney 1976), Data Envelopment Analysis (DEA) (Seiford 1996), ELECTRE (Roy 1968), DELPHI (Dalkey 1963), PROMETHEE (Briggs 1990). The other example of such methodology is Analytic Network Process (ANP) (Saaty 2001) and its simplified variant – AHP (Saaty 2000).

Advantages of AHP/ANP methodology over other commonly used multicriteria methodologies include:

- identical way of treatment of qualitative and quantitative merits,
- simple and readable rules of evaluation,
- possibility of efficient inclusion of uncertainity, risk and fuzziness,
- support for group decision-making,
- high level of automation of computational transformations.

The methodology does obviously has caveats (Kwiesielewicz 2002), but they are continuously eliminated during thanks to introduction of improvements. The indisputable advantage of ANP/AHP is its exhaustive practical verification (Zahedi 1986) and support for multiscenario analysis and taking into account different aspects of analysis, namely: benefits, opportunities, costs and risks (BOCR analysis).

## 2. Municipal heat energy problems

Systems of generation and distribution of heat energy problems are of great importance for cities located in Poland. Because of climate conditions ruling in Poland, Polish cities need continuous supply of energy for heating of buildings up to 7 months a year. Additionally many, especially habitat quarters do need supplies of hot water which is produced thanks to remote heat energy exchange all year long. Thus heat energy production and delivery generates remarkable costs for municipal economy. The heat production and supply systems also influence natural and social environment a lot. Therefore it is very important to make proper short and long time horizon decisions pertaining to construction, maintenance and development of such systems.

The problems of heat energy production and supply systems have been considered for a long time. Utilized methodologies (usually of numerical nature), took into account only technical and economical merits. Present standards of *Sustainable Development* require that influence of social and environmental issues should be taken into account during analysis as well as technical and economical issues.

One of the important problems arising from existence of many feasible alternatives of heat energy source is the choice of most preferable one. The problem is very complex since the choice once made does influence effectiveness of system's operation and whole surrounding environment for a long time. Thus to make appropriate choice multicriteria methodology should be applied.

As an illustration of utilizing of AHP for decision-making, an example pertaining to choice of alternative of heat energy is presented.

## 3. An example of heat energy source choice problem

### 3.1 Problem definition

Set feasible of alternatives for heat energy sources includes:

- coal (ZW),
- natural gas (ZG),
- light oil (ZO),
- renewable sources (ZN).

Following main criteria were included in the analysis:

- financial (investment and operational cost),
- social (local and global),
- economical (local and global),
- protection of surrounding environment (conditions of living, protection of natural environment).

The aim of analysis is to choose the best source of heat energy with regard to considered criteria.

### 3.2 The hierarchy

To two-stage analysis was conducted. Benefits (B) and costs (C) were treated separately and then aggregation of partial results was utilized. Thus two hierarchies of criteria were required.

Analysis of benefits included following main criteria and their subcriteria:

- financial (operational savings BE),
- social (professional activation of local population BS),
- economical (subcriteria: local reliability BGN, energetic safety BGB),
- protection of natural environment (limitation of pollution BN).

Graphical image of criteria hierarchy considered for analysis of benefits is showed in figure 1.

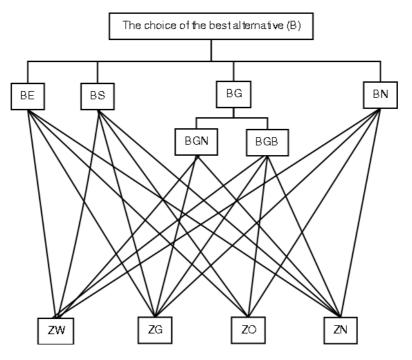


Figure 1. Hierarchy considered for analysis of benefits (B)

Analysis of costs included following main criteria and their subcriteria:

- financial CE (subcriteria: operational cost CEE, investment costs CEI, costs of health protection CEL, costs of appropriate environmental protection level CEN),
- social CS (influence on level of health of population),
- economical CG (cost of energy supply),

• influence on natural environment CN (level of destructive influence).

Hierarchy of criteria considered for analysis of costs is showed in figure 2.

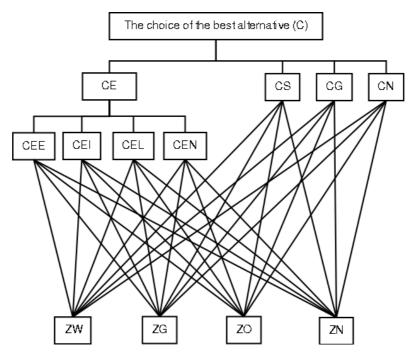


Figure 2. Hierarchy considered for analysis of costs (C)

### 3.3 Analysis of benefits (B)

Priority vectors for main criteria, subcriteria with regard to the aim of analysis (the choice of the best alternative) and heat source alternatives with regard to control criteria (BE, BS, BN) and subcriteria (BGN, BGB) were obtained using right-side eigenvector computed for given form of judgement matrices (Saaty 2000). Numerical computations were made using universal environment for numerical calculations *GNU Octave*, available at http://www.octave.org.

The priority vector for control criteria  $\mathbf{p}_{B}$  was obtained using appropriate judgement matrix shown in table 1 (part of the table surrounded by bold line). Appropriate judgments were obtained using agreed opinions of two experts --- a civil engineer and a heat energy technology engineer. To make judgments Saaty's ratio scale was utilized (Saaty 2000). Maximum eigenvalue  $\lambda_{max}$  and index of consistency i.c. (Saaty 2000) are also presented in table 1.

The financial criterion BE was ranked the most important, while social one (BS) --- the least important. To mark the most important (sub)criteria / alternatives the bold face is used.

Criterion	BE	BS	BG	BN	$\mathbf{p}_{\mathrm{B}}$
BE	1 / 1	7 / 1	5 /1	3 / 1	<i>be</i> = <b>0,5650</b>
BS	1 / 7	1 / 1	1/3	1 / 5	<i>bs</i> = 0,0553
BG	1 / 5	3 / 1	1 / 1	1 / 3	bg = 0,1175
BN	1/3	5 / 1	3 / 1	1 / 1	bn = 0,2622

Table 1. Judgement matrix and priority vector  $p_B$  for control criteria ( $\lambda_{max}$ =4,116; i.c.=0,038)

The local reliability criterion BGN was ranked more important. Priority vectors obtained for subcriteria BGN and BGB of economical control criterion BG (local  $\mathbf{p}_{BG}$  and global one  $\mathbf{P}_{BG}$ ) along with assumed form of judgement matrix are presented in table 2.

Table 2. Judgement matrix and	priority vector obtained for subcriteria	$(\lambda_{\text{max}}=2; \text{ i.c.}=0)$
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Subcriterion	BGN	BGB	P <sub>BG</sub>	$\mathbf{P}_{\mathrm{BG}} = bg \cdot \mathbf{p}_{\mathrm{BG}}$
BGN	1 / 1	3 / 1	0,75	0,0881
BGB	1/3	1 / 1	0,25	0,0294

Similar analysis was made to estimate rank of alternatives with regard to considered criteria and subcriteria. Resulting priority vectors are given in table 3.

 Table 3. Priority vectors obtained for alternatives with regard to considered control criteria and subcriteria (for details about computations see: Appendix 1)

Alternatives	<b>P</b> <sub>ZBE</sub>	<b>P</b> <sub>ZBS</sub>	<b>P</b> <sub>ZBGN</sub>	<b>P</b> <sub>ZBGB</sub>	<b>P</b> <sub>ZBN</sub>
ZW	0,2647	0,1699	0,2323	0,1602	0,0449
ZG	0,0850	0,0996	0,1377	0,0477	0,1348
ZO	0,0552	0,0621	0,0838	0,0986	0,2381
ZN	0,5951	0,6684	0,5462	0,6935	0,5822

Finally, thanks to appropriate aggregation of supermatrix and its transformation (Saaty 2000), final rank of considered alternatives with regard to the aim of analysis was obtained (see: table 4).

Alternatives	P <sub>B</sub>
ZW	0,1959
ZG	0,1024
ZO	0,1073
ZN	0,5944

### Table 4. Final rank of alternatives for benefits analysis

Obtained results show that the best kind of heat energy source with regard to benefits is renewable energy. Other kinds of sources prove to be much, much worse with coal being better than natural gas and light oil.

## 3.4 Analysis of costs (C)

Results of computations for cost analysis are presented in tables 5-8.

# Table 5. Priority vector obtained for control criteria and subcriteria with regard to the aim of analysis ( $\lambda_{max}$ =4,078; i.c.=0,026)

Criterion	СЕ	CS	CG	CN	<b>p</b> <sub>C</sub>
СЕ	1 / 1	4 / 1	6 /1	2 / 1	ce = <b>0,4948</b>
CS	1 / 4	1 / 1	1/3	1 / 5	cs = 0,1336
CG	1 / 6	3 / 1	1 / 1	1/3	cg = 0,0614
CN	1 / 2	5 / 1	3 / 1	1 / 1	cn = 0,3102

# Table 6. Priority vectors obtained for subcriteria with regard to the aim of analysis

 $(\lambda_{\text{max}}=4,056; i.c.=0,018)$ 

Subcriterio n	CEE	CEI	CEL	CEN	<b>p</b> <sub>CE</sub>	$\mathbf{P}_{\rm CE} = {\rm ce} \cdot \mathbf{p}_{\rm CE}$
CEE	1 / 1	5 / 1	4 / 1	2 / 1	0,4869	0,2409
CEI	1 / 5	1 / 1	1 / 2	1 / 5	0,0727	0,0360
CEL	1 / 4	2 / 1	1 / 1	1/3	0,1221	0,0604
CEN	1 / 2	5 / 1	3 / 1	1 / 1	0,3183	0,1575

 Table 7. Priority vectors obtained for alternatives with regard to considered criteria and subcriteria (for details about computations see: Appendix 2)

Alternatives	<b>P</b> <sub>ZCEE</sub>	<b>P</b> <sub>ZCEI</sub>	<b>P</b> <sub>ZCEL</sub>	<b>P</b> <sub>ZCEN</sub>	P <sub>ZCS</sub>	P <sub>ZCG</sub>	<b>P</b> <sub>ZCN</sub>
ZW	0,2041	0,0533	0,4952	0,5822	0,4898	0,5825	0,5670
ZG	0,0843	0,1435	0,1643	0,1348	0,2508	0,0971	0,1479
ZO	0,0502	0,0879	0,2761	0,2381	0,2044	0,2746	0,2370
ZN	0,6614	0,7133	0,0644	0,0449	0,0550	0,0458	0,0481

## Table 8. Final rank of alternatives --- cost analysis

Alternative	P <sub>C</sub>
ZW	0,4499
ZG	0,1420
ZO	0,1871
ZN	0,2210

Results of cost analysis point at coal as the most expensive kind of energy source. The second one is renewable energy source, followed immediately by light oil. The least expensive alternative is natural gas. Obtained rank of alternatives reveales that despite that:

• coal is most commonly available fuel in Poland,

• financial cost of coal is significantly lower than cost of other considered fuels

it is much less preferable with regard to cost level of its utilization to other energy sources. This is mainly because of many undesirable social, economical and environmental issues of coal utilization.

Cost level of utilization of renewable sources for heat production is remarkable but they constitute sources of the cleanest energy. Differences between levels of priorities obtained for renewable energy and natural gas sources are not so big. Thus introducing environmental taxation (it has not been introduced in Poland yet) in short time would make renewable energy even more attractive with regard to cost issues of its utilization.

### 3.5 Aggregation of partial results

To make aggregate assessment weighted aggregation formula (1) was used:

$$P_{i} = b P_{Bi} + (1 - c) P_{Ci}$$
(1)

where *b*, *c* denote weights pertaining to relative importance of benefit and cost analyses,  $P_j$  --- aggregated priority,  $P_{B_j}$  --- benefit analysis priority,  $P_{C_j}$  --- cost analysis priority obtained for considered alternative.

Values of weights b and c estimated using AHP methodology and fundamental judgement scale (Saaty 2000). The values of weights were obtained for different assumptions with regard to relative importance of results of both partial analyses. The formula (1) made it possible to extend the analysis by considering influence of different levels of decision-maker's preferences (table 9).

Table 9. Values of weights b, c obtained for different scenarios	
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Weights	B/C = 1/7	B/C = 1/1	B/C = 7/1
b	0,125	0,500	0,875
С	0,875	0,500	0,125

Final results of computations are given in table 10. Priority vector  $\mathbf{P}$  defines ranks for considered kind of heat energy sources, obtained using formula (1) :

• in case of strong preference for benefits analysis (B/C = 7/1),

• in case of equal preference for both partial analyses (B/C = 1/1),

• in case of strong preference for cost analysis (B/C = 1/7).

Additionally, results obtained for both cases --- analysis of benefits ( $P_B$ ) and analysis of costs ( $P_C$ ) are presented in table 8. Bold entries in table 10. correspond to the best alternatives.

#### Table 10. Aggregated results

Alternative	P <sub>C</sub>	Р			P <sub>B</sub>
		B/C = 1/7	B/C = 1/1	B/C =7/1	
ZW	0,4499	0,1863	0,1865	0,1921	0,1959
ZG	0,1420	0,2776	0,2401	0,1575	0,1024
ZO	0,1871	0,2635	0,2301	0,1564	0,1073
ZN	0,2210	0,2748	0,3433	0,4940	0,5944

Renewable energy constitutes the best kind of heat energy source. This is first of all because of its undeniable advantages. But in case of evident preference for cost analysis preference for other kinds of energy source --- especially natural gas and light oil becomes stronger.

Thus different decisions can be made utilizing the same data, depending on attitude of decision-maker towards particular components of BOCR analysis and existing local conditions.

### 4. Conclusions

Results of conducted analysis confirmed that heat energy source choice is complex task. Commonly used methodologies for solution of this decision problem required introduction of additional simplifications. Application of AHP methodology can considerably ease process of prioritization of feasible alternatives without introducing such simplifications. Thus it is able to provide a decision-maker with accurate, multidisciplinary information about quality of considered alternatives. Besides computational efficiency and simplicity, AHP makes it possible to extend analysis including different scenarios. Such extension makes it easier to include uncertainty and individual preferences of decision-maker during analysis.

## Appendix 1. Details of computations for analysis of benefits

	$(\lambda_{\max}=4,076; i.c.=0,025)$								
Alternative	ZW	ZG	ZO	ZN	<b>P</b> <sub>ZBE</sub>				
ZW	1 / 1	4 / 1	5 / 1	1/3	0,2647				
ZG	1 / 4	1 / 1	2 / 1	1 / 7	0,0850				
ZO	1 / 5	1 / 2	1 / 1	1 / 8	0,0552				
ZN	3 / 1	7 / 1	8 / 1	1 / 1	0,5951				

Table 11. Priority vector  $P_{ZBE}$  obtained for alternatives with regard to control criterion BE

Table 12. Priority vector P <sub>ZBS</sub> obtained for alternatives with regard to control criterion BS
$(\lambda_{\max}=4,057; i.c.=0,019)$

Altternative	ZW	ZG	ZO	ZN	P <sub>ZBS</sub>
ZW	1 / 1	2 / 1	3 / 1	1 / 5	0,1699
ZG	1 / 2	1 / 1	2 / 1	1 / 7	0,0996
ZO	1/3	1 / 2	1 / 1	1 / 8	0,0621
ZN	5 / 1	7 / 1	8 / 1	1 / 1	0,6684

Table 13. Priority vector  $P_{ZBGN}$  obtained for alternatives with regard to subcriterion BGN $(\lambda_{max}=4,051; i.c.=0,017)$ 

Alternative	ZW	ZG	ZO	ZN	<b>P</b> <sub>ZBGN</sub>
ZW	1 / 1	2 / 1	3 / 1	1/3	0,2323
ZG	1 / 2	1 / 1	2 / 1	1 / 4	0,1377
ZO	1/3	1 / 2	1 / 1	1/5	0,0838
ZN	3 / 1	4 / 1	5 / 1	1 / 1	0,5462

Alternative	ZW	ZG	ZO	ZN	P <sub>ZBGB</sub>
ZW	1 / 1	4 / 1	2 / 1	1/6	0,1602
ZG	1 / 4	1 / 1	1/3	1/9	0,0477
ZO	1 / 2	3 / 1	1 / 1	1 / 8	0,0986
ZN	6 / 1	9 / 1	8 / 1	1 / 1	0,6935

Table 14. Priority vector  $P_{ZBGB}$  obtained for alternatives with regard to subcriterion BGB $(\lambda_{max}=4,143; i.c.=0,047)$ 

Table 15. Priority vector  $P_{ZBN}$  obtained for alternatives with regard to control criterion BN ( $\lambda_{max}$ =4,077; i.c.=0,025)

Alternative	ZW	ZG	ZO	ZN	<b>P</b> <sub>ZBN</sub>
ZW	1 / 1	1 / 4	1 / 6	1 / 9	0,0449
ZG	4 / 1	1 / 1	1 / 2	1 / 5	0,1348
ZO	6 / 1	2 / 1	1 / 1	1/3	0,2381
ZN	9 / 1	5 / 1	3 / 1	1 / 1	0,5822

Appendix 2. Details of computations for analysis of costs

Table 16. Priority vector  $P_{ZCEE}$  obtained for alternatives with regard to subcriterion CEE $(\lambda_{max}=4,117; i.c.=0,039)$ 

Alternative	ZW	ZG	ZO	ZN	P <sub>ZCEE</sub>
ZW	1 / 1	3 / 1	5 / 1	1 / 5	0,2041
ZG	1 / 3	1 / 1	2 / 1	1 / 7	0,0843
ZO	1 / 5	1 / 2	1 / 1	1/9	0,0502
ZN	5 / 1	7 / 1	9 / 1	1 / 1	0,6614

Table 17. Priority vector  $P_{ZCEI}$  obtained for alternatives with regard to subcriterion CEI $(\lambda_{max}=4,091; i.c.=0,030)$ 

Alternative	ZW	ZG	ZO	ZN	<b>P</b> <sub>ZCEI</sub>
ZW	1 / 1	1/3	1 / 2	1 / 9	0,0533
ZG	3 / 1	1 / 1	2 / 1	1 / 7	0,1435
ZO	2 / 1	1 / 2	1 / 1	1 / 8	0,0879
ZN	9 / 1	7 / 1	8 / 1	1 / 1	0,7133

Table 18. Priority vector  $P_{ZCEL}$  obtained for alternatives with regard to subcriterion CEL $(\lambda_{max}=4,020; i.c.=0,006)$ 

Alternative	ZW	ZG	ZO	ZN	P <sub>ZCEL</sub>
ZW	1 / 1	3 / 1	2 / 1	7 / 1	0,4952
ZG	1/3	1 / 1	1 / 2	3 / 1	0,1643
ZO	1 / 2	2 / 1	1 / 1	4 / 1	0,2761
ZN	1 / 7	1/3	1 / 4	1 / 1	0,0644

Alternative	ZW	ZG	ZO	ZN	P <sub>ZCEN</sub>
ZW	1 / 1	5 / 1	3 / 1	9 / 1	0,5822
ZG	1 / 5	1 / 1	1 / 2	4 / 1	0,1348
ZO	1/3	2 / 1	1 / 1	6 / 1	0,2381
ZN	1 / 9	1 / 4	1 / 6	1 / 1	0,0449

Table 19. Priority vector  $P_{ZCEN}$  obtained for alternatives with regard to subcriterion CEN $(\lambda_{max}=4,077; i.c.=0,025)$ 

Table 20. Priority vector  $P_{ZCS}$  obtained for alternatives with regard to control criterion CS ( $\lambda_{max}$ =4,156; i.c.=0,052)

Alternative	ZW	ZG	ZO	ZN	P <sub>ZCS</sub>
ZW	1 / 1	3 / 1	2 / 1	7 / 1	0,4898
ZG	1 / 2	1 / 1	2 / 1	4 / 1	0,2508
ZO	1/3	1 / 2	1 / 1	5 / 1	0,2044
ZN	1 / 7	1 / 4	1 / 5	1 / 1	0,0550

Table 21. Priority vector  $P_{ZCG}$  obtained for alternatives with regard to control criterion CG  $(\lambda_{max}=4,121; i.c.=0,040)$ 

Alternative	ZW	ZG	ZO	ZN	<b>P</b> <sub>ZCG</sub>
ZW	1 / 1	6 / 1	3 / 1	9 / 1	0,5825
ZG	1 / 6	1 / 1	1 / 4	3 / 1	0,0971
ZO	1/3	4 / 1	1 / 1	6 / 1	0,2746
ZN	1 / 9	1/3	1 / 6	1 / 1	0,0458

Table 22. Priority vector  $P_{ZCN}$  obtained for alternatives with regard to control criterion CN $(\lambda_{max}=4,061; i.c.=0,020)$ 

Alternative	ZW	ZG	ZO	ZN	<b>P</b> <sub>ZCN</sub>
ZW	1 / 1	4 / 1	3 / 1	9 / 1	0,5670
ZG	1 / 4	1 / 1	1 / 2	4 / 1	0,1479
ZO	1/3	2 / 1	1 / 1	5 / 1	0,2370
ZN	1 / 9	1 / 4	1 / 5	1 / 1	0,0481

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