Multi-criteria optimization of maintenance policy with regard to local infrastructure in case of incomplete information and uncertainty

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1. Introduction

Technical infrastructure systems make it possible to deliver basic services to local population and enterprises.They play, therefore, an important role in case of local communities. On the other hand, they generate considerable costs and that is the reason why their effective operation is of paramount importance.

The effectiveness depends not only on factors controllable by the community. There are also many important non-controllable ones which influence it considerably. These factors include issues pertaining to economic, social and natural environment. There is a problem while addressing them during analysis leading to optimal maintenance policy of the systems.

The main problem with regard to effective coping with these issues is uncertainty or (in the best case) information incompleteness. There are some mathematical tools which make it easier to prepare right decisions in such cases. For example, one can apply methodology based on game theory — games against nature or use multi-scenario approaches — just to mention a few possibilities.

Additionally, many non-controllable factors are of intangible nature. And this makes things even worse. To address this problem adequately the application of special approach is required. Analytic hierarchy process (AHP) [1] method sems to be the best solution in this case. It is shown in the paper how it can be used for assisting decision makers in developing maintenance policy with regard to the operation of community infrastructure systems.

2. Influence of Uncertainty and Information Incompleteness

Community infrastructure systems operate inside complex, diverse environment. The environment is made up of several components. The most important components are economic, social and natural environment ones. Some of them are more, and some less susceptible to the influence of uncertainty and information incompleteness. For example, social environment is commonly perceived as more prone to the uncertainty of behaviour than economic one or natural environment.

The influence of different environmental components on the behaviour of the system in question is of complex interrelational nature. Therefore, the influence of surrounding environment should be perceived as system-wise. This corresponds well with the principle of sustainable development.

One of the most crucial Polish community infrastructure components is municipal district heating and cooling system (MDHCS). It can deliver important services to population all year long. For example, in winter it supplies heat energy for heating dwellings and enterprises and for preparation of hot domestic water. And in summer it is used for cooling purposes. MDHCS system is a good example for community systems, because its operation and maintenance cause substantial costs for a community.

Uncertainty and information incompleteness with regard to MDHCS operation effectiveness is caused by all main surrounding environment components. First of all, social issues matter. For example, energy becomes a fully commercial product in Poland. Its price is rising constantly to cover real costs of its generation. Therefore, the energy usage is highly correlated with the financial state of users and their habits with regard to energy conservation. On the other hand, financial ability to afford energy spendings is strictly linked with the overall state of local and external economy. Social issues can also result these times from migration of people to other Polish regions and even abroad. Therefore, in this case the influence of economic and social environments is highly correlated with each other.

And finally, natural environment can influence operation of MDHCS a lot. First of all, regulations of natural environment protection are evolving. They influence mainly economic issues. Secondly, climate dynamics cause severe threats for effectiveness of the system's operation. Climate conditions, and especially uncertain changes in the value of external air temperature τ_{zc}, decide on optimal values of basic system parameters [3].

Economic issues with regard to energy source / fuel availability play a very important role nowadays [2]. Energy generation in Poland is based mainly on local natural treasure — coal. But because of negative influence on natural environment and economic issues with regard to environmental protection, its application for energy generation is limited constantly. At the beginning of economic transformation in the late 80-ties and early 90-ties light oil and natural gas were perceived as cleaner and safer alternatives than coal. As a result, they became the most popular fuels for energy generation in Poland. But world-wide perturbations concerning their availability during a few last years, prove that that other (economically safer) energy sources, like renewable and nuclear energy, should be utilised. Actual fluctuation of costs is also a problem in case of other, besides energy generation, economic issues.

3. Application of a Game Against Nature

A concept of game against nature comes from the game theory [4]. This is a kind of conflict situation. The first side of the game is the only conscious player — a decision maker. The second side is *a nature* which is not interested in the result of a game. Nature simply expresses conditions (through its *states*) which influence the outcome (*payoff*) of the game. Possible moves of the decision maker are called *strategies*. The strategies correspond to the decisions made. A payoff in a game depends on the decisions and an actual state of nature. A set of payoffs constitutes a so called payoff matrix **A**. The matrix dimensions are: M by N, where M denotes a number of decision maker's strategies and N — a number of different nature states. Therefore, element a_{ij} of **A** denotes a payoff for the decision maker in case of the i-th strategy implementation and j-th state of nature.The payoff matrix is a basis for so called *normal game notation*.

To choose the best decision (payoff ν), a decision maker can use several criteria:

- pessimistic Wald criterion (1);
- extreme optimist criterion (2);
- parametric Hurwitz criterion (3);
- Savage criterion of minimal regret (4);
- Laplace (5) and Bayes criterion (6).

$$\nu = \max_i \left\{ \min_j a_{ij} \right\} \tag{1}$$

$$\nu = \max_i \left\{ \max_j a_{ij} \right\} \tag{2}$$

$$\nu = \max_i \left\{ \gamma \cdot \min_j a_{ij} + (1-\gamma) \cdot \max_i a_{ij} \right\} \tag{3}$$

$$\nu = \min_i \left\{ \max_j \left\{ \max_i a_{ij} - a_{ij} \right\} \right\} \tag{4}$$

$$v = \max_i \left\{ \frac{1}{n} \cdot \sum_{j=1}^{n} a_{ij} \right\} \tag{5}$$

$$v = \max_i \left\{ \sum_{j=1}^{n} p_j \cdot a_{ij} \right\} \tag{6}$$

Symbol *min* (*max*) denotes the choice of the smallest (the biggest) value taken from a column of values (corresponding to decision maker's strategies) in case of symbol *i* placed below (and from row values — corresponding to states of nature — in case of symbol *j* placed below). Parameter γ in (3) takes values from a division $\langle 0;1 \rangle$ and expresses the level of decision maker's relative preference towards pessimism. Symbol p_j in (6) denotes the probability of j-th nature state appearance.

Criteria (1–4) correspond to decision making in case of uncertainty. And criteria (5–6) are used in case of risk situation i.e. incomplete information availability (known probability for the appearance of all states of nature).

4. AHP-assisted Payoff Identification

In order to apply a game against nature, the effects of all decision maker's strategies should be defined for the considered states of nature. When an outcome of strategy implementation is tangible, there is not a problem at all. However, the existence of intangible outcomes makes a problem of payoff estimation much harder. AHP is well suited for joint analysis of tangible and intangible issues. Therefore, it seems that it can help to solve the problem of intangible outcomes with regard to the strategies in the game against nature.

To present the approach, a simple example is provided. A decision maker wants to choose an appropriate strategy for seasonal fuel collection for community enterprise which generates heat energy and supplies it to users in a local area. Three strategies, named A, B, C are considered. Each of them is based on a different fuel supplier. The choice of the supplier results in several consequences for the enterprise. It is assumed that the consequences can be of mixed i.e. tangible-intangible nature. To assess an outcome of the strategy performance, a kind of synthetic measure which includes both tangible and intangible issues is required. AHP is used to measure the effects. Three different scenarios for the changes of external conditions are taken into account. In the first one, mild atmospheric conditions during winter are assumed. The second one is based on the assumption of average conditions. And severe conditions are assumed in the third one. Above scenarios correspond to three states of nature numbered: 1, 2 and 3, respectively.

Due to different conditions of supply, there are differences in the sustainability between considered strategies with regard to possible changes of climate conditions. Thus, the performance of strategy differs from one state of nature to another. To measure the performance of strategies, their rankings can be created in case of every nature state being considered. AHP can deliver normalised weights which describe rankings. Resulting weights can be put into columns of a payoff matrix, corresponding to the considered states of nature. Then, the best strategy can be identified using different criteria of game against nature. The criterion (6) is based on relative probability of states of nature appearance. When levels of probability are not available, they can be estimated using AHP too.

Let us create rankings of strategies in case of considered states of nature.The first step is to collect expert judgments. To estimate normalised weights, additive normalisation method (AN) using 9-point discrete Saaty's scale is applied [1]. The judgments along with obtained results are presented in tab.1. And tab.2 contains AHP judgments and estimated state of nature appearance probabilities p_j. The resulting payoff matrix is presented below (7).

Results obtained using criteria (1–6) are presented in tab.3 (values in parentheses denote the performance indices of strategies). It seems that the best strategy is A, because it appears most frequently as the best one. The second best is B. However, due to criterion (3), strategy B becomes more and more dominant as a pessimistic attitude of decision maker strengthens ($\gamma > 0{,}5$). On the other hand, there are only marginal differences between performances of two top scoring strategies in case of criteria (5, 6) application. Therefore, performance of the strategies can be perceived as more or less equivalent.

Table 1. Estimated rankings of strategies with regard to nature state

	State of nature				State of nature				State of nature			
Strategy	1	2	3	$\mathbf{p}_1$	1	2	3	$\mathbf{p}_2$	1	2	3	$\mathbf{p}_3$
A	1	2	3	0.539	1	2	1/2	0.286	1	1/3	2	0.240
B	1/2	1	2	0.297	1/2	1	1/3	0.143	3	1	4	0.623
C	1/3	1/2	1	0.164	2	3	1	0.571	1/2	1/4	1	0.137
			Sum:	1			Sum:	1			Sum:	1

Table 2. State of nature probability estimation

State of nature	1	2	3	p_j
1	1	1/2	2	0.312
2	2	1	2	0.490
3	1/2	1/2	1	0.198
			Sum:	1

$$\mathbf{A} = \begin{bmatrix} \mathbf{p}_1 & \mathbf{p}_2 & \mathbf{p}_3 \end{bmatrix} = \begin{bmatrix} 0.539 & 0.286 & 0.240 \\ 0.297 & 0.143 & 0.623 \\ 0.164 & 0.571 & 0.137 \end{bmatrix} \quad (7)$$

Table 3. Ranking of strategies due to applied criteria

Criterion	I	II	III
(1)	A (0.240)	B (0.143)	C (0.137)
(2)	B (0.623)	C (0.571)	A (0.539)
(3) $\gamma = 0$	A (0.539)	C (0.413)	B (0.364)
(3) $\gamma = 0{,}25$	A (0.464)	B (0.400)	C (0.397)
(3) $\gamma = 0{,}50$	B (0.436)	A (0.390)	C (0.382)
(3) $\gamma = 0{,}75$	B (0.472)	C (0.366)	A (0.314)
(3) $\gamma = 1$	B (0.508)	C (0.351)	A (0.240)
(4)	A (0.383)	B (0.428)	C (0.486)
(5)	A (0.355)	B (0.354)	C (0.291)
(6)	C (0.358)	A (0.356)	B (0.286)

5. Conclusions

The analysis of some test problems gave encouraging results. Furthermore, the methodology presented is easy for computer implementation. For example, a simple spreadsheet application is enough to make appropriate calculations. Therefore, it seems that the approach constitutes interesting alternative to other methods. It allows to support preliminary decision making quickly and effectively in case of uncertainty and information incompleteness. However, to justify its real-life usefulness with regard to the maintenance of community systems, further research is required.

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